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Hintikka on Information and Deduction

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RESUMEN

Jaakko Hintikka es uno de los más importantes filósofos analíticos de la segunda mitad del siglo XX. Su impacto se percibe fácilmente en muchas ramas de la epistemología actual. Este artículo analiza su distinguida concepción de la información en la lógica deductiva (*información*$_1$). También se describe el modo en que articula su distinción entre información$_1$ de superficie e información$_1$ de profundidad. Mi objetivo es arrojar luz sobre un dilema que el propio Hintikka provocativamente denomina "un escándalo de la deducción". El enfoque que se presenta se centra en el concepto de prueba epistémica y en un concepto diferente de información (*información*$_2$) postulado por la lógica info-teorética tradicional. De este modo se distinguen dos sentidos de 'información' presentes en los estudios lógicos señalándose su naturaleza básicamente complementaria en la explicación de los diversos aspectos involucrados en la práctica deductiva.

PALABRAS CLAVE: *información, implicación lógica, deducción, constituyente, proposición, significado, óntico, epistémico*

ABSTRACT

Jaakko Hintikka is one of the most important analytic philosophers of the second half of the 20th century. His impact is easily felt in many branches of current epistemology. This study discusses his distinctive conception of information in deductive logic (*information*$_1$) and the way he articulates the distinction between surface information$_1$ and depth information$_1$. My aim is to shed light on a dilemma Hintikka himself provocatively calls "a scandal of deduction". My approach focuses on the concept of epistemic proof and on a different underlying concept of information (*information*$_2$) postulated by traditional information-theoretic logic. This enables us to distinguish two senses of 'information' in logical studies by considering their basic complementary nature in explaining the multifaceted aspects involved in deductive practice.

KEYWORDS: *Information, Logical Implication, Deduction, Constituent, Proposition, Meaning, Ontic, Epistemic*

I. Two Complementary Senses of 'Information' in Deductive Logic

The word 'information' is ambiguous in the sense of having different meanings in different contexts. One well-known meaning of 'information' in the realm of deductive mathematical logic is that defined in the Carnap-Bar-Hillel-Hintikka paradigm [see Carnap and Bar-Hillel (1952) and Hintikka (1973/2002)]. In this formal-semantics framework, the concept of information – let us call it *information*$_1$ – is defined through certain extensions of the model-theoretic machinery. Hintikka's concept of information$_1$ turns on surveying constituents – kinds of possible worlds constructed in their characteristic distributive normal forms – which are relative to the means of expression of the language in use. The key idea of Hintikka's constituents ranges from Carnap's state-descriptions of concrete worlds to descriptions of kinds of worlds – the constituents – depicted in the shape of distributive normal forms. In this framework an interpreted sentence is said to contain information$_1$. In a Popperian sense, the information$_1$ contained in a given interpreted sentence is provided by the class of constituents in which the sentence is false. Thus, a tautology lacks information$_1$: it is true in every constituent and hence excludes no constituents. A contradiction contains all information$_1$: it is false in every constituent and hence excludes every constituent. In this framework a premise-set *P* logically implies a conclusion *c*, if and only if, the class of constituents excluded by the conclusion *c* is contained (in the usual set-theoretic sense of 'contained') in the class of constituents excluded by the premise-set *P*. Equivalently, we say that the corresponding premise-conclusion argument-text is *valid*. Given this setting we could call the Carnap-Bar-Hillel-Hintikka information$_1$ approach to logical validity *extrinsic* because it is strictly dependent on the truth-value of the interpreted sentences involved, and "truth" in formal semantics is framed on something external or outside of language, namely the unfolding of constituents, which corresponds to the ways the world is or could have been.

Another sense of 'information' – let us say, *information*$_2$ – is found in the postulates of information-theoretic logic as articulated, for example, in [Corcoran (1998)]. Perhaps the most effective slogans in this conception are that propositions are "carriers of information" and that deduction is "unpacking" the information – more or less hidden, but already contained in the premises. This conception goes back at least as far as the middle of the 19th century, involving thinkers such as Boole, De Morgan, and Jevons, all of whom shared the intuition of an information-based consequence relation. Indeed, this was virtually the dominant conception of logical consequence until the emergence of the Bolzano-Tarski transformation-theoretic paradigm. There are also vestiges of the information-theoretic conception in the 20th century in [Cohen and Nagel (1934/62/93)]. Information-theoretic logic locates logic at the heart of formal epistemology, thereby granting a distinctive

role for our deductive capacities to perform cogent reasoning in the service of knowledge. This viewpoint focuses on mathematical practice, which is often omitted from purely syntactic or semantic accounts of logic. It acknowledges the fact that we routinely determine validity and invalidity relying on our judgment of information containment and non-containment between premises and conclusion in a given premise-conclusion argument. The alternative it provides is genuinely intensional in considering real propositions – not set-theoretic-surrogates of them – and *intrinsic* because propositions are "carriers" of information and hence, contain information. In this picture, a premise-set P logically implies a conclusion c, if and only if, the information in c is already contained in P. Equivalently we say that the corresponding premise-conclusion argument is logically valid. Logical validity is based on a robust sense of 'proposition' involving both information content and logical form. Propositions considered pertain to specific domains of investigation [see Sagüillo (2000)]. A tautology is devoid of information; a contradiction contains all pertinent information. The information contained in a given proposition is provided by the following informational properties of the truth-functional connectives and quantifiers [see Corcoran (2007)]:

1. The information contained in a negation is the information not contained in the proposition that is being negated.
2. The information contained in a conjunction is the information contained in each of its conjuncts.
3. The information contained in a disjunction is the information shared by each of its disjuncts.
4. The information contained in a conditional is the information in the consequent which is not in the antecedent.
5. Every universal's information contains the information of all of its instances – but not conversely.
6. Every existential's information is contained by the information shared by any non-empty set of its instances – but not conversely.

It is worth emphasizing that information-theoretic logic is not based on any extrinsic model-theoretic construction to determine truth-values of interpreted sentences representing information$_1$.The previous postulate-set supports a sense of 'information$_2$' suitable to articulate the objective nature of logical implication while having regard to the epistemic human experience and practice of doing proofs in mathematics. How is it possible that a person or a community of thinkers knows that a certain proposition is true based on previous knowledge of given premises and knowledge of logical implication

between those premises and the obtained conclusion? The suggestion here is that information$_2$ – the concept information-theoretic logic articulates for logical implication – provides a concept which is complementary to information$_1$ – the concept Hintikka proposes with his constituents. Acknowledging the fact that Hintikka has been a champion in studying and developing logical machinery while keeping an eye on our intuitions, the claim is that the picture of logical consequence which information-theoretic logic provides is particularly useful in the context of discovery, where no mechanical procedures are applicable for theoremhood, and mathematicians are "on their own", trusting in their previous knowledge and experience of the specific domain of investigation at the time of trying a proof. We are therefore confronted with two senses of 'information'. Hintikka displays information$_1$ analogously to the way modal logic or epistemic logic displays modal or epistemic notions. The result so obtained is a kind of "applied" logic usually exemplified in so-called philosophical logic. On the other hand, information-theoretic logic is a "pure" logic emanating from the experience of doing concrete proofs in different domains of investigation. It is based on a study of practical deductions that supports a set of postulates for [intensional] propositional information$_2$-content, not an applied logic of information$_1$ based on some construction built upon the model-theoretic machinery.

II. Epistemic Proof in Mathematical Practice and Formal Derivation in Logical Theory

In an important sense, mathematical logic is just an applied branch of mathematics whose aim is to build "artifacts" which model or represent good and bad reasoning [Corcoran (1973)]. Consider, for example, a standard first-order logic < L, D, S >. Its formal system of derivation D codifies in L features of good deduction found in mathematical practice. Likewise, a model-theoretic semantics S codifies features of a conception of validity and invalidity which is also found in practice and which, in turn, sustains the soundness and unsoundness of derivational moves codified in D. Mathematical logic produces such model-artifacts with the material provided by string theory in syntax and by set-theory in semantics. In a way, epistemic or pre-formal proofs are the data for model building a mathematical logic. Therefore, this view of logic as model makes logic appear in some sense empirical. Proofs must exist in order for the logician to be able to study them and to codify their syntactic and semantic features in a formal system. It will not be misleading to say that within the view of logic as model, mathematical logic is instrumental in the sense of being a means towards the understanding of reasoning found in the practice of proof.

In this setting, 'epistemic proof' means a proof which is not the syntactic object that is normally studied in 'Proof-Theory'. It does not pertain to formal semantics or 'Model-Theory', either. More precisely, an epistemic proof is a discourse relative to a person or a community of thinkers, which provides knowledge of a conclusion based on knowledge of the truth of its premises and on the cogency of the chain of reasoning leading from premises to conclusion. It is also worthwhile noting that 'knowledge' here presupposes intelligent beings, but it is taken in its *successful* or incorrigible meaning as found in [Hintikka (1962)]. Looking at the human experience and practice of obtaining knowledge of a true proposition through deduction from premises already known to be true involves – in addition to formal correctness – true content and intelligent beings capable of knowledge. This sense of 'proof' has a perennial and well-established tradition of thought in the history of logic and mathematics. For example, John Myhill [(1960), pp. 461-462] endorsed what he called an absolute sense of proof which was neither syntactic nor semantic but epistemic. This epistemic sense of proof reflects the objectivity of mathematical reality and the dynamic human enterprise of obtaining mathematical knowledge. Similarly, Alonzo Church endorses a strong sense of proof when he says: "Indeed it is essential to the idea of a proof that, to anyone who admits the presuppositions on which it is based, a proof carries final conviction" [(1956), pp. 53-54]. Also John Corcoran [(1989), pp. 22-25] takes proof to be a species of deduction with premises known to be true.

It is important to note that Hintikka uses the expression 'logical inference' as a transformational move made according to a subsystem D of semantic rules in a logical system. This is not to deny that he sometimes presents epistemic desiderata mostly for purpose of illustration, either for enlightening or for qualifying the interest of some of his formal proposals. Moreover, Hintikka uses a particular method of inference, which may properly be called a disproof method, consisting in closing lines of deductive analysis by finding explicit inconsistencies between the truth of the premises and the falsity of the conclusion of a valid argument in each constituent. It should be noted that finding truth-value inconsistencies between premises and conclusion certainly is not unpacking information contained in premises. These two are different epistemic activities involving different capacities.

How do we establish the validity and invalidity of a given premise-conclusion argument in practice? The asymmetry between establishing validity and establishing invalidity of a given premise-conclusion argument has long been noted: validity is established by deduction through an information-processing procedure of reasoning showing that the conclusion follows from the premise-set. A deduction here is a three-part system composed of a premise-set P, an intermediate chain of reasoning R, and a conclusion c. A proof is a deduction with premises all known to be true. On the other hand, the invalidity of a given invalid premise-conclusion argument is established by exhib-

iting a counter-model satisfying the premises, but not the conclusion of the given premise-conclusion argument. The process of establishing validity focuses on information content; the process of establishing invalidity focuses on subject matter and truth-values. The point here is to remind ourselves that, very often, normal mathematical practice seems to sanction epistemic proof – rather than surveying models of a mature axiomatization of a theory – as the usual method to establish logical validity in specific domains of investigation. In this context, the power of information-theoretic logic can be motivated. To begin with, what sort of evidence is a thinker supposed to obtain in order to develop a belief as to whether a given argument-text is valid? Do we survey constituents or models or do we rather check on information containment of the propositions in the argument expressed? Second, how is this experience realized? In particular, what sort of capabilities or skills does a human being deploy when examining information$_1$ associated with constituents? Similarly, what sort of capabilities or skills does a human being deploy when judging information$_2$ containment or non-containment? For the sake of illustration, consider in the universe of the class of the natural numbers the first-order omega argument-text whose premise-set is composed of 'Zero is not successor of zero', 'Successor of zero is not successor of successor of zero', 'Successor of successor of zero is not successor of successor of successor of zero', and so forth, and whose conclusion is the corresponding universal closure 'Every natural number *x* is such that *x* is not its own successor'. Each of the premises is true and the conclusion is true, but the argument-text is logically invalid. A responsible judgment of invalidity may be more accessible from the information-theoretic viewpoint. In this case the previous argument-text is in the same logical form as the next, whose propositions also pertain to the very same class of natural numbers and whose premise-set comprises the following premises: 'Successor of one is not successor of successor of one', 'Successor of successor of one is not successor of successor of successor of one', and so forth. The corresponding conclusion here is the same universal closure 'every number *x* is such that x is not its own successor'. In this second argument-text checking on information-containment almost immediately leads to judgement of invalidity, since this second premise-set lacks one obvious instance, the zero-instance in the established universe. Once the information-theoretic invalidity of this second argument-text is established, the invalidity of the first argument-text is also established through the Principle of Form. The reader should also note that despite the fact that the first argument-text is less obviously invalid than the second, it can also be seen to be information-theoretic invalid, since there is no proposition in its premise-set containing the information that no instance of the universal in the conclusion is left outside the premise-set. In other words, there is no premise to the effect that all of the instances have been taken into account. Of course, constituents in distributive normal form must be finite. The scenario prompted by the omega

example is not accessible and should lead us to no epistemic preference in a virtually infinite search for inconsistency between the truth of the omega premise-set and the falsity of its conclusion.

III. "PARADOX OF INFERENCE" AND "SCANDAL OF DEDUCTION"

Morris Cohen, and Ernest Nagel, perhaps taking pedagogical licence, raised paradoxically the question of how an inference can be informative:

> If in an inference the conclusion is not contained in the premises, it cannot be valid; and if the conclusion is not different from the premises, it is useless; but the conclusion cannot be contained in the premises and also possess novelty; hence inferences cannot be both valid and useful [(1934/62/93), pp. 173-176].

A straightforward way out of this "paradox" is to reclassify it as an ontic-epistemic fallacy. The mistake is confusing the information$_2$ objectively contained in a given premise-conclusion valid argument with the decoded information$_2$ an agent obtains by grasping the propositions in the premise-set and by extracting from it the information$_2$ contained in the conclusion. In a word, the information$_2$-based validity of a given premise-conclusion argument is one thing, and the knowledge of it obtained by deduction or information$_2$ processing in the mind of the thinker is something utterly different. The information$_2$-theoretic concept of logical consequence establishes an ontic or objective relation between a conclusion *c* and a set of premises *P*. Its characteristic *no added information* postulate states that in order for a premise-conclusion argument to be valid it is necessary and sufficient for the information$_2$ contained in the conclusion *c* to be already contained in the premise-set *P*. Using this sense of information it would be [informationally] redundant but perhaps [epistemically] useful to assert *c* in a context in which *P* has already been asserted. What deductive inference does for the agent processing information is to provide *knowledge* that the information in the conclusion is [was] already contained in the premises. It is a logical fact whether a given conclusion *c* is or is not a logical consequence of a set of premises *P*. It is an epistemic fact that, very often, we humans do not know which case it is. Knowing whether *c* is a logical consequence of *P* usually requires considerable intellectual efforts in developing a chain of reasoning *R* showing that the information in **c** is already contained in *P*.

In a way, the so-called "omniscient problem" of contemporary epistemic logic, a well-known feature amply discussed in [Hintikka (1962)] pioneering work, expands the mistake underlying the paradox of inference. Clearly, we are not omniscient, since understanding a premise-set is insufficient to deduce each of its logical consequences. Both puzzles – the paradox of infer-

ence and the omniscient problem – disappear once the objective validity of a premise-conclusion argument is distinguished from deductive knowledge obtained through our limited inferential capacity. The ontic-epistemic distinction coherently settles that *propositions imply* and *people infer*, emphasizing also that logical validity often goes well beyond deductive knowledge of it. One is left with the question of whether these now detected shortcomings of formalism could have been motivated in the first place by a previous conceptual tangle or perhaps by a kind of optimistic way of looking at epistemic logic. The omniscient paradox is solved when recognizing our limitative inferential capacities. Solving the paradox of inference amounts to recognizing what inference does for us despite our limits as thinkers. We come to know that *c* is a logical consequence of *P* when *c* is *deduced* from *P*. Thus, the objective property of validity of a premise-conclusion argument is necessary for deductive information-processing, and deductive information-processing is sufficient for knowledge of logical validity of such an argument. From this perspective, the *no-knowledge* exclusionary view of *inference* and the *all-knowledge* inclusionary view of omniscience are two complementary ontic-epistemic fallacies.

Hintikka perspicuously calls "a scandal of deductive logic" a riddle similar to the paradox of inference:

> Its urgency can be brought home to each of us by any clever freshman who asks, upon being told that deductive reasoning is 'tautological' or 'analytical' and that logical truths have no 'empirical content' and cannot be used to make 'factual assertions': in what other sense, then, does deductive reasoning give us new information? Is it not perfectly obvious that there is some such sense, for what point would there otherwise be to logic and mathematics? This question is apt to cause acute embarrassment, for no such sense has so far been defined in the literature. [(1973), pp. 222-224)]

On this issue Johan Van Benthem (per. comm.) [see also Van Benthem and Martínez (2008), pp. 219-220] points out that conciliation of the objective, let us say, non-ampliative nature of information-theoretic validity with the subjective "amplifying" knowledge of deduction remains even today an open issue of epistemic logic framed within the most sophisticated forms of possible-worlds semantics. The previous quotation from Hintkka prompts the need for a few precisions. The first-order deduction theorem establishes that a deductive inference realized according to transformation rules can be rewritten as a tautological conditional whose antecedent is the conjunction of the premises and whose consequent is the conclusion. This result should not mask the fact that properties belonging to a deduction are not properties belonging to a deduced tautological conditional. Deduction is information processing from premises to conclusion, whereas the corresponding tautological

conditional contains no information$_1$, no process being involved. An unqualified treatment suggests a kind of process-product fallacy in need of correction. The premises and conclusion of a given argument contain information unless they are tautological. If the information in the conclusion is all contained in the premises the argument is valid, otherwise invalid. There is no process involved here. In order to know that a given argument is valid it is sufficient to *deduce* the conclusion from its premise-set. Deduction is information$_2$ processing and presupposes agents. Validity is a property of a premise-conclusion argument (where 'premise' and 'conclusion' are just role-words with no agent involved) which is defined as *no-added information* (in either sense of information). A tautology is a proposition with no information, in either sense of 'information'. Of course, a tautology can be taken to be the limiting case of an argument with the empty premise-set. Since it lacks information, it never adds information to whatever premise-set. Thus, to deduce a tautology amounts to obtaining it from no uncancelled premises. Hintikka seems to endorse the view that the thinker has the capacity to survey constituents to determine information$_1$ and thus, logical validity. Information-theoretic logic holds that the thinker is capable of grasping propositions and of extracting information$_2$ to determine containment or non-containment at each step of a deduction. In agreement with Hintikka, and contrary to positivists, the present viewpoint is that the novelty deduction produced is not purely psychological, but epistemic. The next section provides further discussion and suggests sharpening some elements in Hintikka's own treatment of the issue.

IV. Hintikka's Way out: Surface Information and Depth Information

Hintikka endorses an objective information-concept view when he carefully separates information from epistemic considerations of accessibility to it:

> In another direction, a prima facie plausible view identifies logical inferences with such argument steps that do not introduce any new information into an argument or line of reasoning. ("New" information here means, of course, new to the argument in question, not to the arguer or to the arguer's audience.) [(2007), p. 190)].

This passage says there is one kind of information for logic and that a logical inference (in Hintikka's sense of 'inference') does not add any new information over the information already available. Sometimes Hintikka [(2007), pp. 14-16] expresses the idea by saying that logical inference is not *ampliative*. However, granted that information$_1$ is an objective feature of an interpreted argument-text, Hintikka's final parenthetical sentence above clearly indicates that information can turn out to be epistemically new for the thinker

or for the community of thinkers doing or following the logical inference. Nevertheless, Hintikka [(1973/2002), pp. 22-23] – perhaps inadvertently – blurs the ontic-epistemic distinction already identified as realized in objective information$_1$ and knowledge of it (when available), by postulating two concepts of information$_1$: depth information$_1$ and surface information$_1$:

> …depth information is the totality of information we can extract from a sentence by all the means that logic puts at our disposal. Surface information, on the contrary, is only that part of the total information which the sentence gives us explicitly. It may be increased by logical operations. In fact, the notion of surface information seems to give us for the first time a clear-cut sense in which a valid logical or mathematical argument is not tautological but may increase the information we have. In first-order logic, valid logical inferences must be depth tautologies, but they are not all surface tautologies [(1973), pp. 22-23)].

Hintikka also provides a technical definition of surface and depth information in a given inferential move in his disproof procedure by looking at the syntactic features of the formulas involved. He defines the depth of a given formula as the longest sequence of nested and related quantifiers in it. More precisely, the *degree* of a given formula is the number of the free singular terms of the formula in addition to the number of layers of nested and related quantifiers – its depth. In many cases suitable n-instantiation of quantifiers shall be required in surveying constituents in a given disproof attempt. The n-new layers of quantifiers correspondingly increase the depth of information of the normal form by *n* [see Hintikka (1970)]. Surface information is increased by logical inference. It "recovers" at least some of the depth information for the benefit of the thinker. Sebastian Sequoiah-Grayson (2008) provides a technical discussion of a detailed reconstruction of Hintikka's present proposal, showing the difficulties of the project in providing a general successful solution to the "scandal" once different means of expressions of different languages are considered. The discussion below is intended to raise further philosophical debate sharpening the present viewpoint on the issue.

There is a sense in which the terminology 'depth' and 'surface' *information* appears misleading since it suggests there are two species of information where there is only one. Conceptual confusion may arise if the issue of objective information content is not neatly separate from the subjective issue of what deduction or logical inference does for us, as thinkers. It is an objective feature of a given interpreted sentence that the information$_1$ it conveys is given by the class of constituents it excludes. It is also an objective feature of a proposition that the information$_2$ it contains is given by the class of its logical consequences. It is a subjective feature of an agent whether he establishes or discovers by some inferential procedure whether an objective implication relation does hold. However, *pace* Hintikka, by inference we do not

obtain a new species of information not already contained in premises, much less do we extract a different species of information, namely surface information, out of the other subspecies of depth information in such a way that depth information turns out to be a sort of limit of surface information. I would like to suggest instead that what is really at stake is rather that we discover that the premises contain information for a given conclusion or that the information in the given conclusion is (and was) already contained in the premises [see Sagüillo (2009) for a detailed discussion of this practice-based deduction and Sagüillo (1997) for the corresponding underlying relation of logical consequence]. Once the issue prompted by the scandal of deduction is re-viewed accordingly, the straightforward conceptual way to handle it appears rather neatly: there is one kind of information, and it is some or all of that very same "objective stuff" that is epistemically accessible by each deduction. The next section provides further discussion of this point.

V. Information Content Is Not Meaning

The usual idea of meaning seems to sustain compositionality; i.e., the meaning of each word determines the meaning of the sentence in which they occur. Information$_2$ content, however, is not said to be contained in isolated concepts. Information$_2$, as codified in its sets of conditions, requires propositions or thoughts as the minimum unit for information$_2$ content. Otherwise it would be difficult not to sanction the validity of enthymemes, classically identified as invalid arguments by virtue of their "defective" form. For example, Fred Dretske [(1981), p. 45)] suggests a semantic view in which isolated concepts are said to contain information. This view recovers a philosophical standpoint in which, for example, the concept "man" contains the semantic information of the concept "rational". Thus, according to Dretske's viewpoint, the proposition "Socrates is a man" *implies* "Socrates is rational". It follows that information$_2$ is not meaning since they do not share the same properties.

With a different line of argument, Hintikka endorses the view that information$_1$ is not meaning either. He identifies surface information$_1$ with what a sentence or statement means or "says", and he is right when distinguishing what a person says from what a person "implies" by it. More precisely, what a sentence *says*, the proposition it *expresses*, is not what the proposition so expressed implies, namely the class of its logical consequences – its depth information. Similarly, but certainly differently, what a person asserts, the proposition expressed by the way he uses a sentence on a given occasion, is not what the proposition so expressed implies and, usually, it is not either what the person actually infers from it in that single act. It follows that information$_1$ content cannot be identified with meaning either.

This situation strongly suggests that the terminology 'depth/surface information' appears either redundant, since depth information is just *the* information content of a given proposition – to be identified with the information content in each of its logical consequences – or misleading, since surface information is just the information content of every logical consequence *obtained* from the given proposition by a real or ideal subject by means of logical inference. Again, distinguishing depth and surface information mistakenly suggests a distinction of two species of information. What seems actually to be at stake is an ontic-epistemic distinction which carefully requires a prior landscape of an objective kind of information sustaining logical implication, together with the value of deduction as sufficient for *knowledge* of implication. Information$_1$ is associated with the class of constituents an interpreted sentence excludes, whereas information$_2$ is what propositions contain. These two concepts of information are in an important sense objective. Logical inference is comparing information$_1$ displayed by surveying constituents. Deduction is judging propositional containment and non-containment or information$_2$ processing in the mind of the thinker. These are certainly two different intellectual enterprises with different presuppositions and different aims of inquiry. The present viewpoint, however, suggests that both are complementary.

Further evidence for the viewpoint that information content is not meaning can be provided by considering the context of discovery. In this situation, it is advisable to identify the experiential meaning of a given proposition. Consider, for example, the arithmetical proposition "every perfect number is even". Learning its experiential meaning amounts to operating with numbers and performing calculations with them. This is the kind of human intellectual practice usually discussed in heuristics, as opposed to apodictics. When considering a hypothesis – a proposition not known to be true and not known to be false – it is natural to ask and to look for the kind of evidence that would prompt a belief about the truth-value of the proposition so considered. We may say that a person learns about the objective experiential import of a given proposition – the perfect number hypothesis – by doing a few checks on a few initial perfect numbers – thus generating the subjective experiential bases of a thinker – which may support a conjecture about the truth-value of the hypothesis. Without this initial rational synthetic guess, there is no ground for starting to use rational methods – whether the deductive method or the hypothetic-deductive method – to determine or to come to know apodictically the objective truth-value of the hypothesis. Hintikka [(1973), pp. 23-24] refers to the Kantian constructivist or synthetic view when surveying constituents or model-building in the search of inconsistencies. These two synthetic moves – obtaining experiential bases and surveying constituents – appear to be complementary when confronted with the practice of discovering a proof. We need to have a belief to begin with in order to be able to try deductive methodology to settle a given hypothesis. In order to do

so we need to look at the subject-matter or the "aboutness" of the proposition considered. It is by experiential interaction with numbers and calculations that we elaborate conjectures about a truth-value hypothesis. If the proposition is believed to be true, the natural move is to try to use the deductive method to settle it. The task is to find out a set of premises already known to be true from which the hypothesis is deducible. If the proposition is believed to be false, the natural move is to try to use the hypothetic-deductive method to settle it. The task is to find out a known to be false conclusion deducible from the hypothesis alone or from the hypothesis together with other premises already known to be true [see Corcoran (1989)]. Once more, information content and meaning are different in the present analysis. Both have a certain "objective" import sustaining two different epistemic intellectual exercises. Meaning plays an essential role in obtaining subjective experiential bases to conjecture whether a given hypothesis is either true or false. Information plays an essential role in obtaining "implicants" of the hypothesis in the attempts to use the deductive method, or "implications" of the hypothesis in the attempts to use the hypothetic-deductive method. Both notions are essential to understand epistemically what we thinkers do in practice and without them proper deductive methodology cannot even take off the ground. It seems that formalization and eventual axiomatization require all these previous and prior intellectual practices. On the other hand, Hintikka's intuition seems right when identifying synthetic construction in raising the degree of constituents and taking it as an index of the non-triviality in a particular disproof. My point is not to disregard the fact that information processing in the mind of the thinker and formal constituents surveying are different epistemic moves involving considerations of a different nature. The first appear to be prior to the second and both seem to well serve a complementary picture for the rationale of deductive knowledge.

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