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The Fundamental Problem of Contemporary Epistemology

Ángel Nepomuceno-Fernández, Fernando Soler-Toscano and
Fernando R. Velázquez-Quesada

RESUMEN

Hintikka ha dicho que la abducción es el problema fundamental de la epistemología contemporánea. El propone cuatro tesis (Kapitan-Hintikka) para caracterizar el concepto de abducción, una forma de inferencia al fin y al cabo, por tanto con reglas definitorias y estratégicas. Las observaciones de Hintikka no solo clarifican el modelo clásico de abducción, sino que sugieren modos de aproximación lógica. En este trabajo revisamos todo ello, resumimos el modelo clásico y presentamos una nueva propuesta basada en una perspectiva epistémica.

PALABRAS CLAVE: *abducción, modelo clásico, tableaux, lógica epistémica dinámica.*

ABSTRACT

Hintikka has said that abduction is the fundamental problem of contemporary epistemology. He proposes four theses (Kapitan-Hintikka) to characterize the concept of abduction, a form of inference after all, therefore with defining and strategic rules. Hintikka's remarks not only clarify the classical model of abduction, but also suggest new logical approaches. In this work we review the previous points, summarize the classical model and present a new proposal based on an epistemic perspective.

KEYWORDS: *Abduction, Classical Model, Tableaux, Dynamic Epistemic Logic.*

I. INTRODUCTION

Hintikka has studied the Peircean notion of abduction and qualified it as the central problem in contemporary epistemology. As it is known, Peirce distinguishes three kinds of inference, namely *induction*, *deduction* and *abduction*, which was first called 'explanatory hypothesis' by the American philosopher and must be taken as different from induction and deduction. But, as it has been pointed out in [Martínez-Freire (2010), p. 78], Whewell considers a form of induction that could be taken as a clear precedent of abduction [Whewell (1967)], and Peirce knew the works of the former. In fact, for both authors, Kepler is the best example of the ideal of scientific

method. Despite exegetical discussions, it should be noted, abduction is a different kind of inference.

In the process of constructing scientific theories, a certain reasoning system is adopted: the underlying logic. Abductive triggers appear quite often in science. Sometimes, new facts arise in a way that they are expected to be a consequence of the corresponding postulates, but they are not. Then, new postulates are often added in order to obtain an extended theory that gives account of those new (often surprising) facts. In other cases, a change on the underlying logic may be convenient. In a logical context, given a theory Θ (a set of sentences of a language) and a fact represented by φ (a sentence of the same language) in the framework of the logical system \vdash , the triple $(\Theta, \varphi, \vdash)$ is an *abductive problem* if Θ and φ share some characteristics that, though we expect $\Theta \vdash \varphi$, this is not the case. We may also require that $\neg\varphi$ is not consequence of Θ . We consider an *abductive problem as a triple* $(\Theta, \varphi, \vdash)$ instead of the simple pair (Θ, φ) because we should take into account three parameters, not only the background theory Θ and the fact φ , but also the underlying logic given by \vdash . A *logical explanation* is then another sentence α such that $\Theta, \alpha \vdash \varphi$. Sometimes, it is considered that an abductive solution can be a change of logic instead of an extension of the background theory, and such change may consist on a new set of inferential rules: given the abductive problem $(\Theta, \varphi, \vdash)$, the solution may be a new logic \vdash^* such that $\Theta \vdash^* \varphi$.

The work is organized as follows. We start in the next section by describing how Hintikka interprets Peirce's notion of abduction. The following section is devoted to present the logical treatment of abduction, following mainly the AKM model. Then we study the application of semantic tableaux to search for abductive hypotheses. The last section contains some concluding remarks.

II. WHAT IS ABDUCTION?

In order to give a right answer to that question, several keys can be taken and some points of view are very appropriated. This is the case of Hintikka, who has devoted a paper [Hintikka (1999)] to understand the concept of abduction in a close way to Peirce's intuitions. Let us see how it is defined by the North American philosopher:

Long before I first classed abduction as an inference it was recognized by logicians that the operation of adopting an explanatory hypothesis, – which is just what abduction is – was subject to certain conditions. Namely, the hypothesis cannot be admitted, even as a hypothesis, unless it be supposed that

it would account for the facts or some of them. The form of inference, therefore, is this:

The surprising fact, *C*, is observed;

But if *A* were true, *C* would be a matter of course.

Hence, there is reason to suspect that *A* is true. [Peirce (1998), p. 231]

Hintikka has paid attention to this paper ('Pragmatism as the Logic of Abduction') to analyze the Peircean conception. He considers, in [Hintikka (1999), p. 91], that Peirce's notion (and some of the problems it raises) can be synthesized in the following four theses, which have been first proposed in [Kapitan (1997), pp. 447-478]:

- *Inferential Thesis*. Abduction is, or includes, an inferential process or processes.
- *Thesis of Purpose*. The purpose of "scientific" abduction is both to generate new hypotheses, and to select hypotheses for further examination. Hence, a central aim of such abduction is "to recommend a course of action".
- *Comprehension Thesis*. Scientific abduction includes *all* the operations whereby theories are engendered.
- *Autonomy Thesis*. Abduction is, or embodies, reasoning that is distinct from, and irreducible to, either deduction or induction.

Of course, the four theses should be taken as proposals that are complementary, not mutually exclusive. Hintikka studies the most relevant consequences of the above four theses to facilitate a correct understanding of this fundamental problem of epistemology. Important features pointed out by Hintikka are the presence of a conjectural element in abduction, its ampliative nature and its relation with scientific explanation [Hintikka (1999), pp. 92-93]. The last aspect should be emphasized, since abductive hypotheses should explain the available knowledge in the best possible way. This is why some philosophers have identified abduction with the *inference to the best explanation* (IBE). However, Hintikka objects that with several arguments that can be summarized as follows [Hintikka (1999), pp. 94-96]:

1. The nature of explanation is as difficult to define as the nature of abduction. Explaining an *explanandum* is a complex process.
2. Many of the most important types of scientific reasoning cannot be described as IBE and the appeal to history of science may be

superficial (Newton's theory of gravitation, which could explain not only the motion of planets but also the occurrence of the tides, is not IBE).

3. This point of view is in conflict with the Peircean perspective (in IBE the choice is determined by the facts that are to be explained, in abduction it is not).

From our perspective, IBE does not exhaust the concept of abduction, which is rather a process to obtain *an* explanation in order to clarify something that has been presented as a surprising fact. It is not a simple inference, but a richer process, as the mentioned theses establish. Any discussion about that presupposes the notion of explanation itself. Perhaps IBE could be seen as a metatheoretic idealization of the corresponding processes; then IBE and abduction should not be confused. What about induction-abduction? The autonomy thesis is very clear, abduction is distinct from, and irreducible to, induction. Below we shall see that the idea of "ampliative" inference cannot be applied to justify a possible identification of these notions.

On the other hand, Hintikka points out that Peirce's general notion of inference has a very relevant aspect in order to understand the concept of abduction, namely the relation between premises and conclusion. In other words, an inference can be valid or invalid. Usually, a *rule of inference* is a valid pattern of inference and may be justified in terms of such relation: either 1) the step from the premises to the conclusion is *truth-preserving*, or 2) true premises make the conclusion *probable* to a certain degree. But in abduction, other rules or principles of an altogether different kind must be considered.

To justify an inference, Hintikka proposes two kinds of rules (or principles), in accordance with the known perspective of seeing logical tasks in the form of games: *definitory rules* and *strategic rules*. The former are similar to the ones that define a game like chess: they set the possible moves in a given situation through the game. Strategic rules, on the other hand, tell us which moves are good in order to win the game. In abductive reasoning, both kinds of rules should be considered. We need defining rules to ensure properties like consistency of abductive explanations. But strategic rules are also necessary because of the conjectural element and the ampliative and explanatory character of abduction. Hintikka sees a connection between the Peircean notion and the general theory of questions and answers [Hintikka (1999), pp. 101-103]. Then he brings out an interrogative approach, according to which the difference between ampliative and non-ampliative reasoning becomes a distinction between interrogative (ampliative) and deductive (non-ampliative) steps of arguments.

This consideration of the ampliative character of interrogative steps of arguments gives us more keys to understand the Peircean notion. On the one

hand, abduction is the only way of introducing a new hypothesis in the process of scientific investigation, and it is a process where abductive steps are given (interrogative steps, after all). On the other hand, a theory of scientific explanation is necessary, but, from Hintikka's point of view, this would be equivalent to a study of the logic of why-questions. Then, against the trend of seeing abduction as a form of induction, which would be taken as an ampliative inference, he says that the only justification of an inductive argumentation appeals to the regularity of facts, but this regularity can in principle be refuted by new experiences or discoveries [Hintikka (1999), p. 106].

Finally, Hintikka offers his own solution to the problem of abduction. But there is a double run: themes of the logic of why-questions could lead us to understand the concept of abduction and, at the same time, Peirce's considerations serve as an useful framework to explain some of the main features of the interrogative approach to the methodology of scientific research. Such solution would require a theory of scientific explanation, which cannot forget the logic of why-questions, but this logic, at the same time, depends on the logic of knowledge, that is to say, the epistemic logic, about which Hintikka says that "[epistemic logic]...should more aptly be called the logic of information, the basis of all epistemology should be epistemic logic, suitably developed" [Hintikka (1999), p. 103].

Epistemic logic has been in fact developed so that, after analyzing the classical model of abduction, we shall see another proposal based on the logic of knowledge.

III. THE CLASSICAL MODEL OF ABDUCTION

The classical logical treatment of abduction has been studied in terms of deductive explanation. The starting point is a set of statements (the background theory), and a surprising fact represented by a sentence logically independent of such set. A sentence is considered a solution when together with the background theory implies the surprising fact. This classical model of abduction has been called *AKM-model* in [Gabbay and Woods (2006), p. 49], associated with the names of some of its more visible proponents: Aliseda, Kuipers/Kowalski and Magnani/Meheus. This is a logical approach to the subject, which can be summarized as follows. Given a theory \mathcal{O} , a fact φ , and a logical system \vdash , we say that $(\mathcal{O}, \varphi, \vdash)$ is an abductive problem if and only if it is not the case that $\mathcal{O} \vdash \varphi$. In general, abductive solutions can be found in the following two ways:

- a) Extensions of the background theory (this is properly the case of AKM). This can be done in more than one way, though the most

satisfactory from the perspective of modern philosophy of science may be a reformulation of the “explanatory and consistent” style of abduction studied in [Aliseda (2006), p. 74]. The sentence α is an abductive solution if and only if it verifies the following three requisites:

1. $\Theta, \alpha \vdash \varphi$
2. α is consistent with Θ
3. α does not logically imply φ

b) Change of logic (this case is different but compatible with AKM). Now the logic \vdash^* is an abductive solution if and only if

1. $\Theta \vdash^* \varphi$

The second way can be treated as a different form of abduction, which has been called *structural abduction* in [Keiff (2007), pp. 199-201]. If we concentrate on an abductive process analyzed in terms of a), a question arises, namely, whether such process verifies the above mentioned theses. Through an abductive process two dominant factors are detected: explanationism and consequentialism (distinction due to [Gabbay and Woods (2006), p. 49]). In this model, a consequentialist interpretation of explanations is given, in line with the inferential thesis of abduction. From this position one may identify ‘reasoning’ with the consequence relation of the logical system, \vdash in this context, but while deduction is considered analytic reasoning, abduction is a form of *ampliative* reasoning, so that for Hintikka abductive inferences give (rational) conjectures, that is to say, defeasible conclusions, something that has been insisted on [Gochet (2009)].

The nomological-deductive model of scientific explanation [Hempel and Oppenheim (1948)] can be presented in abductive terms. Basically, the question that arises is that, given a scientific law L (or a set of laws) with the antecedent conditions C , a fact F is explained as a deductive conclusion. In this case the fact F is called *explanans*, while the *explanandum* consists of L and C . The relationship between *explanans* and *explanandum* must be a logical deduction, in this case in the sense of Classical Logic, that is to say $L, C \vdash F$. This model has been extensively discussed, and a general conclusion may be its inadequacy, since a theory of scientific explanation has aspects that cannot be captured by the nomological-deductive model; this could be taken in order to criticize the AKM-model.

Whatever the case may be, in order to determine whether a process is abductive, as it is clear from the previous observations, we may need to confirm that the process is comparable to the AKM model and check that the mentioned theses are present. Likewise, Hintikka’s distinction between

defining and strategic rules of logic would be suitable: the specific rules for abduction are fully strategic. An interesting point here is a discussion about the strategic rules. Of course, in scientific abduction as well as with inferential processes, there are several forms of inferences (induction, statistical reasoning, deduction, etc.), but the abductive steps should be the result of a strategic rule. In the next section we shall try to apply that.

IV. SEMANTIC TABLEAUX

A logical method that can be adopted to treat abduction according to requirements implicit in the AKM model is the semantic tableaux calculus. As it is known, Hintikka's proof method and Beth's tableaux were contemporary. In fact, Beth, Hintikka and Smullyan are the most outstanding authors that have developed this method, with minor differences among them. Beth used tableaux in order to show a counterexample of entailment, but Hintikka introduced the idea of *model set* so that the tableau is a systematic attempt to obtain a model in which a formula $\neg\alpha$ is true, and if such attempt fails, then α is valid. Smullyan has used tableaux as a general method for classical logic. He introduced signed and unsigned tableau systems, all of which are described in [Fitting (1999), pp. 13-23].

The method is an indirect proof procedure (a refutational one, actually) that can be used for several logics but with certain characteristics, and it is a search procedure for models meeting certain conditions. In short, a tableau is a set of sequences of formulas or *branches*, all of which share an initial group of formulas, sometimes called the *root*, where one formula at least is not a literal. Each branch is generated from the root by applications of rules to every non literal formula. When a pair of complementary literals appears in a given branch, then the process stops and the branch is said to be closed; otherwise the process continues until rules have been applied to all non literal formulas.

Semantics establishes that, for a finite set of formulas Γ of a formal language L and a formula β , Γ entails β if all models of Γ are models of β , in symbols $\Gamma \models \beta$. In semantic tableaux, this is just equivalent to the case in which all branches of the tableau for $\Gamma \cup \{\neg\beta\}$ are closed. The current first order system \vdash is sound and complete, so that $\Gamma \models \beta$ is equivalent to $\Gamma \vdash \beta$. Then, the fundamental property of tableaux is a theorem stating that $\Gamma \vdash \beta$ if and only if every branch in the tableau for $\Gamma \cup \{\neg\beta\}$ is closed: this says that it is not possible to simultaneously satisfy Γ and $\neg\beta$.

The logical treatment of abduction has been based on these results. In [Aliseda (2006)], to obtain the set Σ of solutions for the abductive problem $(\Theta, \varphi, \vdash)$, a tableau is constructed with Θ and $\neg\varphi$ as the root, and then Σ has

been defined as the set of formulas that close the tableau. The abductive method works in this way: a formula $\alpha \in \Sigma$ is an abductive solution if and only if $\Theta, \alpha \vdash \varphi$, which is the case if and only if the tableau of $\Theta \cup \{\alpha, \neg\varphi\}$ is closed. The problem of semidecidability that arises with first-order formulas has been treated in [Nepomuceno-Fernández (2002)], where DB-tableaux have been used, a kind of tableaux where the δ -rule has been modified such that from the formula $\exists x\beta$ the current branch is divided into $n+1$ (the number of constants that occurred before plus one) branches that continue with formulas $\beta(a_1/x), \dots, \beta(a_n/x), \beta(a_{n+1}/x)$, instead of the standard rule according to which from $\exists x\beta$ the same branch continues with the formula $\beta(a_{n+1}/x)$, for the new constant a_{n+1} (with a_n the last constant that occurred in the branch).

Let us see a first example at propositional level. The theory Θ is $\{p \rightarrow q, \neg q \vee r\}$ and the fact φ is $r \vee s$. In order to look for α such that the tableau $T(\Theta, \alpha, \neg\varphi)$ is closed, we construct $T(\Theta, \neg\varphi)$. The only open branch has the formulas $\{p \rightarrow q, \neg q \vee r, \neg(r \vee s), \neg r, \neg s, \neg q, \neg p\}$, so the set of (possible) solutions is formed by literals that close this branch, namely $\{r, s, p, q\}$. If we rule out r and s (as they are trivial solutions), then both p and q are the unique (non trivial) solutions. Observe how, strategically, the solution p is more relevant, as the derivation of the fact from p involves more formulas of the theory than the derivation from q . It can be seen by comparing the corresponding natural deductions:

1. $p \rightarrow q$	Premise (theory)
2. $\neg q \vee r$	Premise (theory)
3. p	Premise (solution)
4. q	<i>Modus ponens</i> (1,3)
5. $\neg\neg q$	Double negation (4)
6. r	Disjunctive syllogism (2,5)
7. $r \vee s$	\vee -Introduction (6)

1. $p \rightarrow q$	Premise (theory)
2. $\neg q \vee r$	Premise (theory)
3. q	Premise (solution)
4. $\neg\neg q$	Double negation (3)
5. r	Disjunctive syllogism (2,4)
6. $r \vee s$	\vee -Introduction (5)

Let us see a very simple example with first order formulas: let the theory be $\Theta = \{\forall x\exists y(Rxy \wedge \neg Rxx)\}$ and $\varphi = Rba$. The corresponding DB-tableau is displayed as follows

$$\begin{array}{c}
 \forall x \exists y (Rxy \wedge \neg Rxx) \\
 \neg Rba \\
 \exists y (Ray \wedge \neg Raa) \\
 \exists y (Rby \wedge \neg Rbb) \\
 \begin{array}{cc}
 / & \backslash \\
 Raa \wedge \neg Raa & Rab \wedge \neg Raa \\
 Raa & Rab \\
 \neg Raa & \neg Raa \\
 X & \dots \\
 & Rbc \\
 & \neg Rbb \\
 & \exists y (Rcy \wedge \neg Rcc) \\
 & Rca \\
 & \neg Rcc
 \end{array}
 \end{array}$$

The tableau contains more branches that are not represented in order to abbreviate. At the right we see a complete open branch that could be closed by adding the literals Rba , $\neg Rab$, Raa , $\neg Rbc$, Rbb , $\neg Rca$ or Rcc . Here Rba is the unique trivial solution (the above requisite 3 of the classical model fails). Others like Raa , Rbb and Rcc are not consistent with the theory (requisite 2). Some of the literals that close the branch leave other open branches, so we only take $\neg Rbc$ as good solution, which together with the theory entails Rba in all universes with three individuals. Now deduction (in finite frames) has to take into account the number of constants that consistently instantiate the matrix of the initial formula. The derivation would be:

- | | |
|--|------------------------------|
| 1. $\forall x \exists y (Rxy \wedge \neg Rxx)$ | Premise (theory) |
| 2. $\neg Rbc$ | Premise (solution) |
| 3. $\exists y (Rby \wedge \neg Rbb)$ | \forall -instantiation (1) |
| 4. $(Rba \wedge \neg Rbb) \vee (Rbb \wedge \neg Rbb) \vee (Rbc \wedge \neg Rbb)$ | \exists -instantiation (3) |
| 5. $\neg (Rbb \wedge \neg Rbb)$ | Prop. tautology |
| 6. $\neg (Rbc \wedge \neg Rbb)$ | Prop. reasoning from 2 |
| 7. $Rba \wedge \neg Rbb$ | From (4), (5), (6). |
| 8. Rba | |

The use of a DB-tableau instead of a standard one is motivated by the fact that the standard tableau generates an infinite branch. However, the DB-tableau shows that the initial set of formulas, which has the finite model property, is satisfiable in a domain with just three elements. Then by reasoning in such kind of domains, if we have the initial formula and $\neg Rbc$, a correct conclusion is Rba , which is in consonance with the available information,

according to which the first instantiation is with the constant b , since the second premise is a negative literal and such instantiation must be with respect to its first argument position. Now the definitory rules have been specified through the construction of the fragment of the DB-tableau (or each deduction), but as the abductive procedure has been considered, it contains strategic elements (for example, the use of DB-tableaux, how to choose the best abducible among the obtained ones, etc.).

Tableaux can also be applied to work with structural abduction. In [Nepomuceno-Fernández, Salguero and Fernández (2012)] tableaux for (normal) modal logics, with specific rules for the corresponding accessibility relation, show that from rules (exclusively) for a logic K , the schema $\Box\varphi \rightarrow \varphi$ is not valid, because the only branch of its negation is open, but it is enough to add reflexivity as new rule to close such branch. That is to say, the method shows the transition from logic K to logic T , from logic T to logic $S4$, and so on. Since determined prefixes of modal operators can generate branches with infinite elements, by modifying the rule for the possibility operator, new tableaux, which are correlates of DB-tableaux, can be defined.

Resolution can be also used, just as semantic tableaux, to study logical abduction. Now the goal, displaying suitably the involved formulas, is to discover what is necessary to construct a valid formula; then the notion of clause is based on formulas that are in disjunctive normal form instead of conjunctive normal form (it should be noted that, in classical terms, to every valid reasoning corresponds a logical principle, a universally valid formula), that is, the calculus is a “dual” clausal calculus. In [Soler-Toscano and Nepomuceno-Fernández (2006)] an algorithm based on that has been designed and in [Soler-Toscano, Nepomuceno-Fernández and Aliseda (2009)], resolution and finite tableaux have been combined.

On the other hand, as the inferential parameter of abduction is the underlying logic, tableaux for non classical logics should be taken into account, despite the use of mentioned modal tableaux for modal logic. In fact, tableaux for modal and multimodal logics have been studied, for example in [Fitting and Mendelsohn (1998)] and in [Priest (2008)] and they could be applied to treat abduction when the underlying logic is of that kind, *mutatis mutandis*. Sometimes scientific inquiry may find inconsistent theories, without implying an immediate rejection of the theory as a whole. This case has been studied in [Carnielli (2006)], where the starting point is a critic of the AKM-model for its inability to tackle abduction in such cases. However, new modifications of tableaux allow to settle a logic for formal inconsistency and the methods for searching solutions to abductive problems do not change in the most important things.

Do these models respond in an acceptable way to the problems of modeling processes of abduction? Do they verify the mentioned Kapitan-Hintikka's theses? A short revision could give us the answer. The main problem

may arise with respect to the thesis of purpose, since the tableaux provide a set of formulas, but some choices not indicated by the specific rules of tableaux must be done. Perhaps choices belong to a set of strategic rules, since the best explanation is actually a maximum aspiration. Whatever the case may be, some functions could be defined, based on the history, according to which a good choice may be the one that involves more formulas of the theory to obtain the solution, or any other that could be considered, but, in the last resort, that would not be a defining rule but a strategic one.

V. THE EPISTEMIC PERSPECTIVE

From a logical perspective, the classical definitions of abductive problem and abductive solution only mention a theory and a formula. However an epistemic and dynamic approach to abductive reasoning is possible, which has been suggested by Hintikka, as we mentioned above. Dynamic epistemic logic can be seen as one of the most productive frameworks for developing any philosophy of information, so that the question of tackling the problem of searching solutions for abductive problems can be seen as a problem of obtaining certain information.

The genesis of dynamic epistemic logic has been summarized in [van Ditmarsch, van der Hoek and Kooi (2008), pp. 3-10], where the basic notion of information is given as something that is relative to a subject (an agent, but there can be more). This subject, thus, has certain perspective on the world, but this perspective can change due to communication. Certain questions, as the following, arise

1. What is an abductive problem from an agent's information point of view?
2. What is an abductive solution in terms of the actions that modify the agent's information?
3. Do these notions change when we explore different kinds of agents?

In [Soler-Toscano and Velázquez-Quesada (2014)] these and other questions have been studied. Let us see that in short. In order to treat logically abduction, we use now notation in a dynamic epistemic logic style. The agent's perspective should be taken into account. Let Φ be the information that is available for her; then the agent has a *novel χ -abductive problem* whenever neither χ nor $\neg\chi$ are part of her information (in symbols, $\neg\text{Inf}\chi \wedge \neg\text{Inf}\neg\chi$), and she has an *anomalous χ -abductive problem* whenever $\neg\text{Inf}\chi \wedge \text{Inf}\neg\chi$. By using a dynamic style, solutions to these kind of abductive problems can be defined as follows:

1. With respect to a novel χ -abductive problem: a formula ψ is a solution if ψ can be added to the agent's information in a way such that χ is part of the agent's information afterwards, which can be expressed in symbols as $\langle \text{Add } \psi \rangle \text{Inf } \chi$
2. For anomalous χ -abductive problem: a formula ψ is a solution if $\neg\chi$ can be removed from the agent's information in such a way that, after that, $\langle \text{Add } \psi \rangle \text{Inf } \chi$ is the case, in symbols $\langle \text{Rem } \neg\chi \rangle \langle \text{Add } \psi \rangle \text{Inf } \chi$.

According to the style of abduction, we shall have specific conditions. The formula ψ is a *consistent* abductive solution if $[\text{Add } \psi] \neg \text{Inf } \perp$ is the case, with \perp representing the falsehood and $[\text{Add } \psi] \alpha$, defined as $\neg \langle \text{Add } \psi \rangle \neg \alpha$, indicating that α will be the case after ψ is added in any way. The formula ψ is an *explanatory* abductive solution if there are β_1, \dots, β_n such that $\langle \text{Rem } \beta_1 \rangle, \dots, \langle \text{Rem } \beta_n \rangle [\text{Add } \psi] \neg \text{Inf } \chi$ (with $[\text{Rem } \psi] \alpha$ defined as $\neg \langle \text{Rem } \psi \rangle \neg \alpha$). The formula ψ is a *minimal* abductive solution if, for every other φ , $([\text{Add } \varphi] \text{Inf } \chi \wedge [\text{Add } \psi] \text{Inf } \varphi) \rightarrow [\text{Add } \varphi] \text{Inf } \psi$.

It should be taken into account that in logical approaches a strong assumption is made, since a theory is usually assumed to be closed under logical consequence, so the agent's information is closed under logical consequence, i.e., we have an *omniscient* agent, which not only knows the postulates of a theory but all their (logical) consequences. But this is not the current situation neither in scientific inquiry nor in real life. Then we should make a difference between what the agent actually has, her *explicit information* (InfEx), and what follows logically from it, her *implicit information* (InfIm). If so, a χ -abductive problem appears when χ is not part of the agent's explicit information. There is a natural relation between implicit and explicit information, as $\text{InfEx } \varphi \rightarrow \text{InfIm } \varphi$.

Several combinations of such notions of information with the corresponding negations and (affirmed and denied) formulas are possible. Then, more precise formalizations of novelty and anomaly can be presented. Specifically:

1. The *truly novel case*: $\neg \text{InfEx } \chi \wedge \neg \text{InfEx } \neg \chi \wedge \neg \text{InfIm } \chi \wedge \neg \text{InfIm } \neg \chi$.
A solution is a formula ψ such that $\langle \text{Add } \psi \rangle \text{InfEx } \chi$. A solution can also be a formula ψ and a reasoning step α , represented as $\langle \alpha \rangle$, such that $\langle \text{Add } \psi \rangle (\text{InfIm } \chi \wedge \langle \alpha \rangle \text{InfEx } \chi)$
2. The *truly anomaly case*: $\neg \text{InfEx } \chi \wedge \text{InfEx } \neg \chi \wedge \neg \text{InfIm } \chi \wedge \text{InfIm } \neg \chi$.
A solution now has two steps, first a contraction to remove $\neg \chi$,

$\langle \text{Rem}\neg\chi \rangle (\neg\text{InfEx}\chi \wedge \neg\text{InfEx}\neg\chi \wedge \neg\text{Inflm}\chi \wedge \neg\text{Inflm}\neg\chi)$ and then it can be solved as 1.

Dynamic epistemic logic offers a natural framework to model many of the subjective features of abductive reasoning [Velázquez-Quesada, Soler-Toscano, Nepomuceno-Fernández (2013)]. Indeed, it is possible to define strategic rules to select the agent's best explanation [Nepomuceno-Fernández, Soler-Toscano, Velázquez-Quesada (2013)].

VI. CONCLUDING REMARKS

The most important characteristics of abduction are represented by the mentioned theses (due to Kapitan and Hintikka), so any logical treatment of abduction should take them into account. This is the case of tableaux method (and resolution), a procedure in which two kinds of rules can be distinguished, definitory and strategic rules. This is a crucial distinction introduced by Hintikka, who emphasizes that “the true justification of a rule of abductive inference is a strategic one” [Hintikka (1998), p. 111]. In fact, as we have pointed out, strategic elements appear when tableaux are used to capture abductive processes.

The epistemic perspective, which has been summarized, was first defended by Hintikka since he considers that epistemic logic is the basis of epistemology. Then the use of dynamic epistemic logic to study abduction is consistent with Hintikka's point of view, so that the mentioned proposal becomes a natural continuation of Hintikka's suggestion and a new tool for analyzing (complex) abductive processes. Usually agents are assumed to be omniscient, but real agents have different abilities. This is another circumstance that is taken into account in approaches based on this logic. Finally, combinations of formulas and reasoning to solve an abductive problem may be a new step in unifying a treatment of standard and structural abduction, which may be very necessary, specially if abduction, as Hintikka says, is the fundamental problem of contemporary epistemology.

Grupo de Lógica, Lenguaje e Información

Universidad de Sevilla

Camilo José Cela, s/n. 41018 Sevilla. Spain

E-mails: nepomu@us.es, fsoler@us.es and FRVelazquezQuesada@us.es

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