Modelling Linguistic Context with Hintikka Sets and Abduction

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RESUMEN

Los conjuntos Hintikka pueden verse como descripciones parciales de un mundo posible y usarse para construir modelos formales en una lógica modal para la interpretación de las expresiones del discurso en el marco de un contexto o una situación concretos. Para ello, definimos los contextos lingüísticos mediante conjuntos Hintikka como un sistema de marcos que permite al hablante/oyente asignar referencia, por ejemplo, a cualquier expresión anafórica de un modo general, tomando el mejor candidato como referente en cada fragmento de discurso para darle una interpretación.

PALABRAS CLAVE: *Conjuntos Hintikka, tablas semánticas, significado lingüístico, abducción, lógica de contextos*.

ABSTRACT

Hintikka sets can be viewed as partial descriptions of a possible world and used to construct formal modal models for interpretation of discourse expressions in a concrete context or situation. Linguistic contexts are accordingly defined by means of Hintikka sets as a system of frames that allows the hearer/speaker to assign a reference, for instance, to any anaphoric expression in a general way, picking up the best referential candidate for each fragment of discourse to get its interpretation.

KEYWORDS: *Hintikka Sets, Semantic Tableaux, Natural Language Meaning, Abduction, Context Logic*.

I. SOME PRELIMINARY WORDS

A general theory of meaning should provide interpretation models of linguistic expressions that go beyond the boundaries of traditional semantics. Meaning in natural language depends, in a very important manner, on syntactic aspects of speech (both functional and relational aspects) and pragmatic aspects, which are also involved in the correct interpretation of linguistic utterances. So it is a well-known fact, for instance, that many natural language expressions must be interpreted in the context in which they appear, and that

this context can sometimes be ambiguous and dependent on non-referential structures such as morphological concordance or lexical relations among terms. One of the most evident cases is the interpretation of anaphoric pronouns (Pro) and noun phrases (NP):

> (1) John told Susan that my novel was on sale in a downtown bookstore and she bought it.

In this sentence we have only two fully referential expressions (if we consider that proper names always denote the same individuals): John and Susan; two NPs that can get their reference in a broader context ("my novel", which requires knowledge of the denotation of the common noun as well as the reference of the speaker, and "a downtown bookstore", that, in addition to the denotation of the common names involved, requires a spatial context to place it); and two personal pronouns "she" and "it" that need to be related to any of the terms appearing in the context (or other non-contextual, but situational terms). So we have the following instantiation possibilities for the anaphoric pronouns:

$$\|she\|=|\{\|John\|, \|Susan\|, \|my\_novel\|, \|a\_downtown\_bookstore\|\}$$

$$\|it\|=|\{\|John\|, \|Susan\|, \|my\_novel\|, \|a\_downtown\_bookstore\|\}$$

The arguments for rejecting some of these possibilities have been given at some length in [Salguero and Soler (2010)]. In a nutshell, we do not consider the NPs of this sentence as good candidates for the reference of the pronoun "she" because of the assignation of thematic roles by the verb "bought":

$$\|she\|\neq\|my\_novel\|$$

$$\|she\|\neq\|a\_downtown\_bookstore\|$$

But we do consider them candidates for the pronoun "it":

$$\|it\|=\|my\_novel\| \text{ or }$$

$$\|it\|=\|a\_downtown\_bookstore\|$$

However, we still have several options for assigning reference to both pronouns that English speakers would rule out. The first one is obvious:

$$\|she\|\neq\|John\|$$

The reason here is morpholexical gender instead of thematic roles: "she" is a feminine pronoun, but John is a masculine name. Then we have just one candidate in the context:

||she||=||Susan||

All right, but what about the reference of the pronoun "it"? What do we interpret as bought by Susan, when we try to make sense of the sentence (1): my novel or a downtown bookstore?

In order to get a functional model for interpretation of sentences like (1), we have to deal with the problem of formalizing context. It is evident that we need to build a context where we would be able to assign the correct reference to the pronoun "it" in the example. Both possibilities are real: maybe Susan bought my novel; maybe she bought the whole bookstore (an eccentric, but possible behaviour). Can an English speaker logically decide on the best interpretation? Can we infer from the context which is the best candidate for the reference of the pronoun "it"?

Our proposal consists in an adaptation of Hintikka sets to *Modal First Order Logic* which will let us define linguistic context as a set of interpretation frames. We will apply a semantic tableaux procedure to get models that satisfy a certain type of quantified sentences. These models will then allow us to describe a set of contexts where anaphoric instantiation problems could be resolved, as well as to define criteria for choosing from a number of given contexts the best one for anaphora interpretation by means of abduction. For this, a definition of a contextual metalogic for dealing with this procedure will be given.

II. HINTIKKA SETS

A Hintikka set μ (also known as a *model set*) is a set of formulas of a language $\mathcal{L}$ that satisfies the following conditions [Hintikka (1969)]:

1. $\perp \notin \mu$
2. For every wff α if $\alpha \in \mu$ then $\alpha \notin \mu$
3. For every wffs α and β:
    a. If $\alpha \wedge \beta \in \mu$ then $\alpha \in \mu$ and $\beta \in \mu$
    b. If $\alpha \vee \beta \in \mu$ then $\alpha \in \mu$ or $\beta \in \mu$
    c. If $\alpha \rightarrow \beta \in \mu$ then $\alpha \notin \mu$ or $\beta \in \mu$

This is the simplest way to define Hintikka sets for a propositional logic. Of course, we may deal with predicates as well as with propositions. For that, we

must extend the notion to a *First Order Language* $\mathcal{L}_{FOL}$ with identity, in which we have to define a set of individual constants or parameters *Cons*, such that for every wff α and any individual constants $a, b \in Cons$:

1. If $\alpha(a) \in \mu$ and $(a = b) \in \mu$, then $\alpha(b) \in \mu$

2. If $\exists x\ \alpha(x) \in \mu$ then $\alpha(a) \in \mu$ for at least one individual constant α appearing in the formulas of μ, that is to say: $\alpha \in D(\mu)$

3. If $\forall x\ \alpha(x) \in \mu$ then $\alpha(a_1 \ldots a_n) \in \mu$ for every individual constant $a_1 \ldots a_n$ appearing in the formulas of μ: $\{a_1 \ldots a_n\} \subseteq D(\mu)$

A Hintikka set can be viewed as a partial description of a *state of affairs*, in the sense this notion was defined by the philosopher Rudolf Carnap in [Carnap (1947)]. Intuitively, the terms "state of affairs" and "possible world" are synonymous. Both concepts refer directly to a particular description of the reality. Hence, in the first approximation, we can define a state of affairs as a course of events and facts which include objects of our discourse and that is referred to by our statements. Following this definition, the so-called *Indian Chronicles* about the colonization and conquest of America are different descriptions of a state of affairs (or several different ones), *The History of the Decline and Fall of the Roman Empire* by Edward Gibbon describes a state of affairs of the ancient world, as well as the novels by García Márquez describe different states of affairs of Macondo, an invented city, or even Newton's *Principia Mathematica* is a representation of a supposedly eternal and immutable state of affairs of the Universe.

But this definition brings us closer to the concept of *possible world*, since we conceive it as the referent – or better yet, the *descriptum* – of a coherent set of sentences of a given language, a set that cannot be increased by the addition of a new sentence without becoming inconsistent. And this is precisely the essence of any Hintikka set.

Such a definition of possible worlds allows the comparison from a logical point of view of a variety of consistent sets of statements, and also lets us determine the truth of any of these in terms of, not only the current state of affairs, but also other possible courses of events. That is to say, a statement can be interpreted as far as it is known what to expect about the world in the case it is true.

To manage this idea, it is necessary to define a complementary notion to the concept of Hintikka sets: *model class*. A model class is a non-empty set of Hintikka sets that describe a collection of related possible worlds. Let us call this model class Ω. We can define a relation $\Re$ on the Hintikka sets in Ω such that $\Re \subseteq \Omega^2$ [Hintikka (1969), p. 61]. Hence, a *model system* (S-model) is defined as a 3-tuple $<\Omega, \Re, D>$, where Ω is a model class, $\Re$ is the acces-

sibility relation defined in Ω and D is a function defined on Ω such that for any $\mu \in \Omega$, $D(\mu) = \{\alpha \mid \alpha \in Cons\}$, i.e.: a non-empty domain of individuals.

Now we can extend Hintikka sets conditions to a FOL with modal operators:

1. If $<\cdot> \alpha \in \mu \in \Omega$ then there is at least one Hintikka set $\nu \in \Omega$ such that $\mu \Re \nu$ and $\alpha \in \nu$

2. If $[\cdot] \alpha \in \mu \in \Omega$ then for every $\nu \in \Omega$ such that $\mu \Re \nu$, $\alpha \in \nu$

It is important not to mix up S-models and interpretations: a S-model is a frame for interpreting the formulas of a certain number of Hintikka sets, which implies a difference with Kripke's approach to modal logic through possible worlds as actual models for a formula.

It is well known that different definitions of the relation $\Re$ yield different S-models for different modal logics. For example:

S-models	Properties of the relation $\Re$	Clauses for the relation $\Re$
K-model	-----------	-----------
KD-model	Seriality	$\forall \mu \exists \nu \, (\mu \Re \nu \ \& \ \mu \neq \nu)$
T-model	Reflexivity	$\forall \mu \, (\mu \Re \mu)$
B-model	Reflexivity	$\forall \mu \, (\mu \Re \mu)$
	Symmetry	$\forall \mu \, \nu (\mu \Re \nu \Rightarrow \nu \Re \mu)$
S4-model	Reflexivity	$\forall \mu \, (\mu \Re \mu)$
	Transitivity	$\forall \lambda \mu \nu \, (\lambda \Re \mu \ \& \ \mu \Re \nu \Rightarrow \lambda \Re \nu)$
S5-model	Euclidianity	$\forall \lambda \mu \nu \, (\lambda \Re \mu \ \& \ \lambda \Re \nu \Rightarrow \mu \Re \nu)$

All these concepts allow us to define the logical notions of satisfiability and validity in terms of a concrete S-model used as the frame for interpreting the formulas of our language:

- A formula α is satisfiable in an S-model (S-satisfiable) iff there exists a Hintikka set μ such that $\alpha \in \mu \in \Omega$

- A formula α is S-valid (briefly $\vDash_s \alpha$) iff for every Hintikka set $\mu \in \Omega$, $\alpha \in \mu$

We can also establish the following corollary from Hintikka's and Lindenbaum's lemmas and the concept of Hintikka set:

Hintikka's Lemma: Every Hintikka set for $\mathcal{L}_{\text{FOL}}$ over a non-empty domain D is satisfiable in a structure whose domain is D.

Lindenbaum's Lemma: Any consistent theory of predicate logic can be extended to a complete consistent theory.

Corollary: Every Hintikka set μ is a subset of a maximally consistent set in a semantic sense.

III. WHAT CAN SEMANTIC TABLEAUX DO FOR US?

In a nutshell, a semantic tableau $T_s\alpha$ is a sequence of sequences of formulas derived from a first formula α by means of some rules defined as counterparts of the conditions of a Hintikka set and the definition of a certain S-model. We say that a semantic tableau is closed when all of its sequences contain a pair of formulas of the form β and $\neg\beta$. Otherwise, we say it is open. Open semantic tableaux provide a method for constructing Hintikka sets from its open sequences. Or what is the same: semantic tableaux provide models for interpreting any consistent set of formulas. This is possible thanks to the following lemmas and the *fundamental theorem of semantic tableaux*:

Lemma 1: For every formula α and every open sequence $\Sigma \subset T_s\alpha$ there is an S-model that simultaneously satisfies all the formulas in the sequence (this lemma can be demonstrated by induction on the set of model sets in Σ).

Lemma 2: For any S-satisfiable formula α there is at least one open sequence $\Sigma \subset T_s\alpha$ such that an S-model that simultaneously satisfies all the formulas in the sequence Σ can be found (this lemma can be demonstrated by induction on the length of a given sequence $\Sigma \subset T_s\alpha$).

Theorem (the fundamental theorem of semantic tableaux): $T_s\alpha$ is closed iff α is not S-satisfiable.

The *Fundamental Theorem of Semantic Tableaux* can be proved appealing to the preceding lemmas in the following way [Salguero (1991), p. 95]:

If a semantic tableau $T_s\alpha$ is not closed then there is at least one open sequence $\Sigma \subset T_s\alpha$. By Lemma 1, if $\Sigma \subset T_s\alpha$ is open then it is possible to find an S-model that satisfies all the formulas in $\Sigma \subset T_s\alpha$. Hence, α is S-satisfiable and by contraposition if α were not S-satisfiable then $T_s\alpha$ would be closed.

On the other hand, if α is S-satisfiable then by Lemma 2 there must be a sequence $\Sigma \subset T_s\alpha$ whose formulas are simultaneously S-satisfiable. But in case $\Sigma \subset T_s\alpha$ was closed, it would occur only because there is a pair of formulas of the form β and $\neg\beta$, which means that the formulas in $\Sigma \subset T_s\alpha$ would not be simultaneously S-satisfiable. Therefore if $\Sigma \subset T_s\alpha$ is closed then α is not S-satisfiable.

In order to apply semantic tableaux as a way to get models for sentences of a Modal First Order Logic (MFOL) we must define a method for identifying individuals in the different Hintikka sets provided by the tableaux. In other words, we need a logical counterpart of Kripke´s *crossworld identification* in Hintikka S-models. We will call it *individuating functions*.

An individuating function is a way to recognize an individual across contexts. It is the formal translation of our natural capacity to identify the same object in different contexts and situations [Hintikka (1969); (1975)]. Quantifying over individual variables in opaque contexts – such as, for example, modal contexts – requires existence of individuating functions that guarantee rigid reference for individuals in all the possible worlds described by a class of related Hintikka sets. Hence, what is guaranteed if we can define a function f such that $f(x, D(\mu_1)) = f(y, (D(\mu_2)) \ldots = f(z, D(\mu_n))$ – where x, y, … z are any individual variables in the sentences of a class of accessible Hintikka sets $\{\mu_1, \mu_2, \ldots \mu_n\}$ and $D(\mu_1), D(\mu_2), \ldots D(\mu_n)$ are their respective domains – is the existence of rigid designators in our semantics. This kind of function is defined as follows:

The set Φ of individuating functions is a set such that for every function $f \in \Phi$ and every pair of domains $D(\mu)$ and $D(\nu)$, if $f(x, D(\mu)) = a$ and $f(y, D(\nu)) = b$ and $\mu\Re\nu$, then if $(a = b) \in \mu$ it is also the case that $(a = b) \in \nu$. So defined this set can be seen as a subset of the set of those intensional functions that Carnap called "individual concepts" [Carnap (1947), pp. 39-42].

In [Salguero (1991)] the rules for construction of semantic tableaux for MFOL were defined. There we used the notion of S-model to describe the different logics, and the following definition to characterize them in terms of their referential capabilities:

An S-model is a fixed domain model iff for every $\mu, \nu \in \Omega$, if $\mu\Re\nu$ then $D(\mu) = D(\nu)$. Otherwise, it is a variable domain model.

It is clear that a fixed domain model guarantees recognition of any individual in any accessible world, but it is also evident that it is not possible to restrict ourselves to S-models of this kind. For instance, we can consider the Barcan Formula:

[BF]: $\forall x \, [\cdot] \, \alpha(x) \rightarrow [\cdot] \, \forall x \alpha(x)$

As is well known, [BF] is valid in symmetric logics, that is to say: it is S-valid in those S-models where the accessibility relation is defined as symmetric, namely B-models and S5-models. In this respect, we can also prove the following:

$\models_s [BF]$ for every fixed domain S-model.

But the fact is that [BF] is not S-valid in non-symmetric S-models since its validity depends on symmetry, as was first established in [Hughes & Cresswell (1968), pp. 173-174]. Hence we propose the following conjecture:

Conjecture: Every non-symmetric S-model is a variable domain model.

If this was the case, we could consider fixed domain models as a special case of variable domain models, since fixed domain models can be obtained by adding [*BF*] to non-symmetric S-models as an axiom (v. gr.: T+[*BF*] or S4+[BF]). So, we can define two types of variable domain models:

- The S-models that satisfy the *nested domains requisite*: $\forall \mu, \nu \in \Omega$, $\mu \Re \nu \Rightarrow D(\mu) \subseteq D(\nu)$.
- The S-models that do not satisfy the *nested domains requisite* (Krippkean free modal logics), where it is possible to find two Hintikka sets $\mu, \nu \in \Omega$ such that $\mu \Re \nu$ and $\alpha \in D(\mu)$ but $\alpha \notin D(\nu)$.

Fixed domain models are the limit case of nested domain models. In the latter models, the converse Barcan Formula

[BF*]: $[\cdot] \, \forall x \alpha(x) \rightarrow \forall x \, [\cdot] \, \alpha(x)$

is S-valid, but not the Barcan Formula [BF] itself, except in the limit case $D(\mu) = D(\nu)$.

Therefore, [*BF*] implies that domains cannot grow when we move from the actual world to another accessible possible world, which means that all objects existing in accessible possible worlds also exist in the actual possible world, but this is counterintuitive with respect to the fact of communicative increment in discourse, for example.

On the other hand, [BF*] states that domains cannot shrink when we move from the actual world to other accessible possible worlds, which is

congruent with the fact that new states of information preserve the possibility of existence of previously known objects.

These considerations – together with the impossibility of expressing existence in the usual sense in Kripkean free modal logics – impose nested domains S-models as the best candidates for modelling natural language and discourse interpretation. Therefore, the notion of an S-model we need for our purposes is the following:

> **Definition:** An S-model is a 4-tuple $<\Omega, \Re, D, f>$, where $\Omega, \Re, D$ are defined as before and $f \in \Phi$ is a function such that for every two Hintikka sets $\mu, \nu \in \Omega$, if $\mu\Re\nu$ then $f(x, D(\mu)) \in D(\nu)$ for any individual variable x.

Now, we can exemplify using models by means of semantic tableaux applied to MFOL. Let us consider this tableau for an instance of the negation of [BF]:

$$\neg\big(\forall x[\cdot](Px \rightarrow Qx) \rightarrow [\cdot]\forall x(Px \rightarrow Qx)\big)/1_{\neg}$$
$$\forall x[\cdot](Px \rightarrow Qx)/1_a$$
$$\neg[\cdot]\forall x(Px \rightarrow Qx)/1_{\neg}$$
$$[\cdot](Pa \rightarrow Qa)/1_{1,2}$$
$$\langle\cdot\rangle\exists x\neg(Px \rightarrow Qx)/1_2$$
$$(Pa \rightarrow Qa)/1_{\rightarrow}$$
$$\exists x\neg(Px \rightarrow Qx)/2_b$$
$$(Pa \rightarrow Qa)/2_{\rightarrow}$$

| $\neg Pa/1$ | | $Qa/1$ | |
$\neg(Pb \rightarrow Qb)/2_{\neg}$		$\neg(Pb \rightarrow Qb)/2_{\neg}$	
$\neg Pa/2$	$Qa/2$	$\neg Pa/2$	$Qa/2$
$Pb/2$	$Pb/2$	$Pb/2$	$Pb/2$
$\neg Qb/2$	$\neg Qb/2$	$\neg Qb/2$	$\neg Qb/2$

Each open sequence in the previous tableau provides a model that satisfies $\neg[BF]$, where the numerical indices mark the number of related Hintikka sets needed for the model. Notice that the new parameter b appearing in a formula marked with the index 2 does not mark the formula $\forall x [\cdot] (Px \rightarrow Qx)/1$. This means we have assumed that the accessible relation is not symmetric. Then we have four non-symmetric models for $\neg[BF]$. This is the leftmost one:

$\Omega = \{\mu_1, \mu_2\}$ such that $\mu_1, \Re\mu_1, \mu_2\Re\mu_2, \mu_1\Re\mu_2$

$D(\mu_1) = \{a\}, D(\mu_2) = \{a, b\}$ and $f(x, D(\mu_1)) = a$ and $f(x, D(\mu_2)) = a$

S4-model	Pa	Pb	Qa	Qb	$Pa \to Qa$	$Pb \to Qb$
μ_1	0	---	---	---	1	---
μ_2	0	1	---	0	1	0

Of course, as said above, when the tableau is not open we have no model that satisfies the initial formula, which means that its negation is valid. This is the way to prove the validity of [BF] in a symmetric model, for example in an S5-model:

$$\neg(\forall x[\cdot](Px \to Qx) \to [\cdot]\forall x(Px \to Qx))/1_\neg$$
$$\forall x[\cdot](Px \to Qx)/1_{a,b}$$
$$\neg[\cdot]\forall x(Px \to Qx)/1_\neg$$
$$[\cdot](Pa \to Qa)/1_{1,2}$$
$$\langle\cdot\rangle\exists x\neg(Px \to Qx)/1_2$$
$$(Pa \to Qa)/1_\to$$
$$\exists x\neg(Px \to Qx)/2_b$$
$$(Pa \to Qa)/2_\to$$

$\neg Pa/1$			$Qa/1$		
$\neg(Pb \to Qb)/2_\neg$			$\neg(Pb \to Qb)/2_\neg$		
$[\cdot](Pb \to Qb)/1_{1,2}$			$[\cdot](Pb \to Qb)/1_{1,2}$		
$\neg Pa/2$		$Qa/2$	$\neg Pa/2$	$Qa/2$	
$Pb/2$		$Pb/2$	$Pb/2$	$Pb/2$	
$\neg Qb/2$		$\neg Qb/2$	$\neg Qb/2$	$\neg Qb/2$	
$(Pb \to Qb)/1_\to$		⋮	⋮	⋮	
$(Pb \to Qb)/2_\to$		⋮	⋮	⋮	
$\neg Pb/1$	$Qb/1$	⋮	⋮	⋮	
$\neg Pb/2$	$Qb/2$	⋮	⋮	⋮	
X	X	X	X	X	X

It is easy to see from the above examples that the use of individuating functions ensures models whose domains are minimal, avoiding an unnecessary multiplication of entities when we are interpreting any satisfiable formula of MFOL. This is even more evident when we have an iteration of modal operators and quantifiers over non-monadic predicates, because we can find some infinite tableaux that could be closed imposing certain restrictions – as

in the case of Díaz-Boolos tableaux – that are related to these individuating functions [Salguero (1991), pp. 166-172].

<div style="text-align:center">IV. CONTEXT ABDUCTIVE LOGIC</div>

We have just seen how open semantic tableaux are able to provide us with models that satisfy a given MFOL formula. These models are based on Hintikka sets rather than on Kripkean semantics. In this sense, Hintikka sets can be viewed as partial descriptions of possible worlds, and we may use them as contexts or frames where it is possible to give an interpretation of a sentence or a sequence of sentences in relation with other contexts or frames. For example, given a sentence in natural language, it could be the case that its anaphoric variable expressions (v.gr. pronouns) have more than one possible interpretation in more than one context. For instance, the sentence we analysed in the first section:

   (1) John told Susan that my novel was on sale in a downtown bookstore and she bought it.

This sentence consists of three simpler sentences, one of them being a subordinate clause and the other two main clauses. The natural way to interpret the anaphoric pronouns "she" and "it" in the last sentence is in connection with the first main sentence and its subordinate clause. As we have explained, it is obvious that we have at least two possible contexts for the interpretation of the pronoun "it". Those contexts are related to the interpretation of the first sentence. So it would be very useful for us to treat the interpretation of this sentence on a different level than the interpretation of the sentence "she bought it". That is to say, we need to see the models of interpretation of "John told Susan that my novel was on sale in a downtown bookstore" as contexts for the interpretation of "she bought it". Then, interpreting the first sentence as introducing modal contexts, we can have a set of models (at least two) and a set of contexts for the interpretation of the variable "it", i.e.:

$$f\,(it, D\,(\mu)) = \|\ my\ novel\ \|$$

$$f\,(it, D\,(\nu)) = \|\ a\ downtown\ bookstore\ \|$$

To do this, we need a modal context logic, acting as a metalogic regarding MFOL and the model systems defined using Hintikka sets. This logic for contexts is based on the proposal by Guillaume Aucher, Davide Grossi, Andreas Herzig and Emiliano Lorini of a dynamic context logic in [Aucher et al. (2009)], as well as the application of this logic to scientific inference by Ig-

nacio Hernández, Ángel Nepomuceno and Enrique Sarrión in [Hernández et al. (2012)].

Let a contextual model (C-model) be $M = <W, R, \mathfrak{I}>$ such that:

1. $W \neq \varnothing$ is the set of Hintikka sets that describe some possible states or situations.

2. For each $\mu \in \Omega$, $R(\mu) \subseteq W$, i.e.: a context is thought as a set of states.

3. $\mathfrak{I}$ is a valuation such that for every sentence a, $\mathfrak{I}(a) \subseteq W$. $\mathfrak{I}(a)$ represents the set of states that satisfy a.

The context logic L_c is defined in the following terms:

$$\varphi := \alpha \in MFOL \mid \neg\varphi \mid \varphi \to \psi \mid [\mu]\,\varphi$$

where $\mu \in \Omega$ and $[\mu]\,\alpha$ expresses that in the context defined by the Hintikka set μ, it is the case that α. We define $[\Omega]$ as a global operator such that $R(\Omega) = W$. We can also define the operator $<\mu>$ in the following way: $<\mu>\varphi = \neg[\mu]\neg\varphi$, expressing that in the context defined by μ the sentence φ is possible.

The semantics of L_c is defined as follows, as in [Hernández et al. (2012), pp. 22-23]. Given a C-model $M = <W, R, \mathfrak{I}>$, for any $w \in W$:

1. For every $\alpha \in \mu \in \Omega$, $M, w \vDash_c \alpha$ iff $w \in \mathfrak{I}(\alpha)$

2. $M, w \vDash_c \varphi$ iff $M, w \nvDash_c \neg\varphi$

3. $M, w \vDash_c \varphi \to \psi$ iff $M, w \nvDash_c \varphi$ or $M, w \vDash_c \psi$

4. For any $\mu \in \Omega$, $M, w \vDash_c [\mu]\,\varphi$ iff $M, w \vDash_c \varphi$ and for every $w' \in R(\mu)$, $M, w' \vDash_c \varphi$

In every state, "φ holds in a context" is equivalent to say that the actual state and all the states in the context satisfy φ:

$$M, w \vDash_c [\mu]\,\varphi \text{ iff } R(\mu) \subseteq \mathfrak{I}(\varphi)$$

Saying "φ holds in some state of a context" is equivalent to say that some state that satisfies φ is in the context:

$$M, w \vDash_c <\mu>\,\varphi \text{ iff } \mathfrak{I}(\varphi) \cap R(\mu) \neq \varnothing$$

Hence $\vDash_c [\mu] \, \varphi \rightarrow <\mu> \varphi$ is an immediate consequence of the definition, i.e.:
$[\mu] \, \varphi \rightarrow <\mu> \varphi$ holds for every C-model.

As can be seen, the logic L_c is a reinterpretation of Hintikka model systems in which Hintikka sets are considered to define contexts. These contexts can be interpreted, in turn, as theories about what is and what is not, including relevant presuppositions of existence and hence the possible referents of those individual variables that appear in the evaluated formulas. So, we can use L_C to "elaborate" contexts of interpretation when we have non-instantiated individual variables, as in the case of anaphora. To do this, it is an interesting strategy to define a logical calculus where some abductive inferential rule is included in addition to the deductive ones. The following is a simple abductive calculus for context logic:

1. All the axioms and theorems of MFOL

2. $[\Omega] \, \varphi \rightarrow \varphi$

3. *Modus Ponens*

4. Necessitation rule (respect to Ω): if $\vdash_c \varphi$ then $\vdash_c [\Omega]\varphi$

5. Peirce's rule or abduction rule: for every $\mu \in \Omega$, if $\vdash_c [\Omega] \, (\psi \rightarrow \varphi)$ and $\vdash_c [\mu]\varphi$ then $\vdash_c <\mu> \psi$

Peirce's rule allows inferring the possibility of adding a formula in a given context if that formula implies in the model another one that appears necessarily in the context. This means that in a given context we can have individuals that appear in other contexts of the model, provided that there are certain relationships between formulas. Of course, this is very interesting in order to apply individuating functions through contexts.

If we also define a consequence relation on the basis of these inference rules, we will have an abductive context logic as an extension of a normal modal predicate logic. For instance, let $M = <W, R, \Im>$ be a context model and let the formulas φ and ψ be free of modal operators. Then for a context $R(\mu) \neq \varnothing$ the following relation is defined:

$$\varphi \Rightarrow^*_\mu \psi \text{ iff } M \vDash [\mu]\varphi \rightarrow <\mu> \psi$$

This relation $\Rightarrow^*_\mu$ defines a logic for every model M valid for non-empty contexts. This restriction is desirable since the most interesting contexts for interpretation of individual variables in discourse are those where we can have a reference for the individuating functions, of course.

V. FINAL REMARKS

We have seen how Hintikka sets provide us with a way of describing partial worlds in a semantic manner. The notion of model systems based on Hintikka sets is also a well-known tool to work with different types of modal logics, including modal first order logics (MFOL). These concepts permit application of semantic tableaux methods for getting interpretation models of sentences, including those sentences containing individual variables in a MFOL.

Pursuing this idea, we have described a metalogic that operates over Hintikka sets as though they were contexts, and, what is more interesting, we have defined it as a modal logic, too. This logic of contexts has been enriched with an abductive rule that generates new possible information in the context in a non-random manner, and yields an inference relation.

The abductive context logic, as defined here, may help to select those more suitable contexts for interpretation of natural language sentences that include anaphoric variables. It is a way of context elaboration, a central notion in the dynamic interpretation of a series of discourse phenomena, as noted in [Aliseda (1997)]. In this sense, the interpretation of any utterance can be understood as a search by the listener for the best possible explanation that makes it true (or at least that makes it compatible with the previous information).

Of course, abduction can be seen as a method for constructing this explanation, the context in which all the individual variables lacking reference in a single sentence or proposition can be instantiated. From this point of view, abductive context logic provides the appropriate tool for generating any information which is not explicitly present in a given inferential process, but which is necessary for this process to be carried out correctly.

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