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Inferential Knowledge and Incremental Confirmation

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In *Scepticism and Reliable Belief* (*SRB* hereafter) José Zalabardo urges that inferential and non-inferential knowledge should receive separate accounts. According to him a paradigm case of inferential knowledge is testing whether a liquid is an acid. If the litmus paper is dipped in it and it turns red, we conclude that it is. We *know*, indeed, that the liquid is an acid because we have *adequate evidence* – we can see the colour of the litmus paper – for it. Inferential knowledge, notwithstanding, must satisfy two sorts of conditions. Objective conditions concern the link between the propositions at issue – "ACID" and "RED", for short. Subjective conditions, in contrast, have to do with the relationship between the epistemic agent and RED, and also to her relationship to the connection between RED and ACID. The goal of this paper is to discuss Zalabardo's account of objective conditions for inferential knowledge.

I. Objective Conditions for Inferential Knowledge.

According to Zalabardo the objective constraint on inferential knowledge can be summarized as follows:

> *S* cannot know *H* on the basis of *E* unless *E* provides *adequate support* to *H*.

Initially he suggests two necessary conditions for adequate support (*SRB*, sec. 4.3):

C1. $0.5 < b < p\,(H \mid E) < 1$ (where b is a real number higher than 0.5)

C2. a low value for $p\,(E \mid \neg H)$

The first condition excludes the possibility of getting inferential knowledge on the basis of *E* when $\neg H$ is as probable at least as *H*.[1] The second condition discards situations where the rate of *false-positives* is non-negligible. Let us suppose that the litmus paper turned red in many cases when the liquid is not an acid, fifty per cent for instance, so $p\ (E|\neg H) = 0.5$. Could RED provide adequate support for ACID even though $p\ (E|H) \approx 1$ − that is, even though if the liquid is an acid, then the paper almost invariably turns red? Zalabardo claims that if *E* is adequate evidence for *H*, *E* should be unlikely to obtain were *H* false. Then, a significant rate of false-positives would defeat *E*'s support for *H* and, consequently, inferential knowledge of *H* on the basis of *E*.

Although those two conditions are necessary for adequate support, Zala bardo acknowledges that they are not sufficient given that both conditions can be simultaneously fulfilled even though *H* and *E* are probabilistically independent, that is, even if $p\ (H|E) = p\ (H)$. It seems clear, however, that *E cannot support H unless E has any positive effect on H's probability*. So, it is evidence that raises *H*'s probability that is relevant here. According to this, the so-called "Bayesian relevance criterion" states a minimal necessary requirement for all *incremental confirmation* measures:

E confirms *H iff* $p\ (H\ |E) > p\ (H)$

E disconfirms *H iff* $p\ (H\ |E) < p\ (H)$

E has no confirmational effect on *H iff* $p\ (H\ |E) = p\ (H)$

It is worth mentioning here that this is a *qualitative* constraint for incremental confirmation, insofar as it gives a yes/no answer to the question whether a particular bit of evidence *E* confirms hypothesis *H*. The aim of a quantitative account of confirmation is, nonetheless, *measuring* the confirmational impact of evidence. However, a myriad of mathematical functions that satisfy the Bayesian relevance criterion have been suggested for calculating the exact degree to which a hypothesis is (dis)confirmed by the evidence. In *SRB* section 4.4 Zalabardo distinguishes between "two leading approaches": the "probability approach" and the "likelihood approach". Since definitions for the degree of confirmation usually consider either differences or ratios, there are four different measures:[2]

	Differences	Ratios
Probabilities	$PD(H,E) = p(H \mid E) - p(H)$	$PR(H,E) = \frac{p(H \mid E)}{p(H)}$
Likelihoods	$LD(H,E) = p(E \mid H) - p(E \mid \neg H)$	$LR(H,E) = \frac{p(E \mid H)}{p(E \mid \neg H)}$

All of these measures fulfil the Bayesian relevance criterion and agree on qualitative assessments, that is, on whether or not *E* (dis)confirms *H*. Nevertheless, they generally disagree on the exact amount to which *E* confirms *H*. Quantitative disagreement is not really disturbing here. Since there is no fact of the matter concerning which are the measured units, there is no fact of the matter about which result gives us the objective reading, so to say. But quantitative disagreements occasionally give way to different orderings of pairs evidence/hypothesis. Given that two measures, $\boldsymbol{S_1}$ and $\boldsymbol{S_2}$, are ordinally equivalent *iff* for all *H*, *E*, *H'* and *E'*, it is true that:

$$\boldsymbol{S_1}(H, E) \geq \boldsymbol{S_1}(H', E') \text{ iff } \boldsymbol{S_2}(H, E) \geq \boldsymbol{S_2}(H', E')$$

it has been proved that the aforementioned measures are not *ordinally equivalent*.[3]

The question, then, is which measure should be preferred. Zalabardo resorts to intuitions and argues that they favour *LR* (we will discuss his argument in the next section). In particular, inferential knowledge demands a high value for *LR*. Since favouring *LR* makes condition C2 redundant – a high value for any of those confirmation measures that compare likelihoods, i.e., *LD* and *LR*, entails a low rate of false-positives–, it is replaced by

C2* a high value for $\dfrac{p(E/H)}{p(E/\neg H)}$

Now, C1 and C2* are necessary conditions – jointly sufficient – for claiming that *E* provides *adequate support* to *H*. Insofar as *S* could not know *H* on the basis of *E* unless *E* provided adequate support for *H*, them, if any of both conditions were not fulfilled, inferential knowledge would be frustrated.

II. Intuitive Grounds for Confirmation Measures

Intuitions favour *PR* over *PD* and *LR* over *LD*, according to Zalabardo, so the final choice is between *PR* and *LR*.[4] Given that $\dfrac{p(H|E)}{p(H)}$ is equivalent to $\dfrac{p(E|H)}{p(E)}$, and $p(E) = p(E|H) \cdot p(H) + p(E|\neg H)\, p(\neg H)$,[5] it is easy to see that the disagreement between *PR* and *LR* concerns the weight given to the rate of false positive results at their respective denominators.[6] Zalabardo puts forward an adequacy condition for confirmation measures that, supposedly, is intuitively grounded:

AC If $p(E|H) = p(E^*|H^*)$ and $p(E|\neg H) < p(E^*|\neg H^*)$, then *E* confirms *H* to a higher degree than *E** confirms *H**.

Yet *AC* is always fulfilled by *LR* but only sometimes by *PR*.

It is worth explaining here why this difference arises. If the antecedent of *AC* is true, it follows from *LR* that the rate of false-positives alone decides which of both hypotheses is more confirmed by the evidence. According to *PR*, in contrast, initial probabilities of the hypotheses at issue could counterbalance their false-positives rate in such a way that the hypothesis with a lowest rate is less confirmed than the alternative hypothesis. It is true that *PR* necessarily satisfies *AC* if $p(H) = p(H^*)$, that is when priors are "neutralized". But *PR* gives a bonus to low initial probability as long as positive confirmation is at issue and, conversely, the effect of disconfirmatory evidence is smaller for those hypotheses with higher probability.[7] Now, since $p(E)$ and $p(E^*)$ depend on the initial distribution of probability – recall that $p(E) = p(E|H) \cdot p(H) + p(E|\neg H) \cdot p(\neg H)$ − not only likelihoods, but priors are also crucial for discerning whether *E* confirms *H* to a higher degree than *E** confirms *H**. Particularly, even though the antecedent of *AC* is true, *E** could *PR*-confirm *H** to a higher degree than *E* confirms *H* when $p(E^*) < p(E)$, in contrast to the verdict of *LR*. Here is an example. Let $p(E|H) = p(E^*|H^*) = 0.8$, $p(E|\neg H) = 0.2$, and $p(E^*|\neg H^*) = 0.4$. Thus, $LR(H, E) = 4$ and $LR(H^*, E^*) = 2$. Therefore, $LR(H, E) > LR(H^*, E^*)$, and *AC* is fulfilled. Now, let us suppose that $p(H) = p(\neg H) = 0.5$, but $p(H^*) = 0.1$ and $p(\neg H^*) = 0.9$. Then, $p(E) = 0.5$, $p(E^*) = 0.44$, $PR(H, E) = 1.6$ and $PR(H^*, E^*) \approx 1.82$. Consequently, $PR(H, E) < PR(H^*, E^*)$ and *AC* is violated.

Zalabardo claims that "...a plausible theory of confirmation should take same true-positive ratio with a lower false-positive ratio as a sufficient condition for a higher degree of confirmation, even when we are dealing with different hypotheses." (*SRB*, 82) But, why should we accept *AC*? Zalabardo's intuitive example takes *E* = wheezing, *H* = asthma, *E** = loss weight, *H** = lung cancer. Now, the rate of true-positives is high in both situations, so $p(E|H) \approx p(E^*|H^*)$. But the rate of false-positives is significantly different, so $p(E|\neg H) << p(E^*|\neg H^*)$, since many people who do not have lung cancer also lose weight. When asked about this example, Zalabardo points out that we intuitively consider that wheezing confirms asthma to a higher degree than weight loss confirms lung cancer. It is worth noticing, however, that this example is fine provided that $p(H)$ and $p(H^*)$ are more or less the same. Since I have discussed Zalabardo's argument for *AC* in a joint paper,[8] here I will focus instead on some further alleged advantages of *LR* over *PR*.

(a) *Priors Are Not Well Defined*

Zalabardo states that the field covered by *LR* is wider than that covered by *PR* since there are many cases where conditional probabilities are well defined but the corresponding unconditional probabilities are not (*SRB*, 83).

Some remarks are in order here. Firstly, he takes for granted that unconditional probabilities are involved when calculating *PR*. In fact, *PR* resorts to *p* (*H*) or, in an equivalent formulation − see above footnote 5−, to *p* (*E*). However, Bayesian advocates of confirmation measures insist that all probabilities included in Bayes' Theorem are conditional probabilities. According to Bayesian accounts of learning, the "initial" probability distributions of the epistemic agent are not taken from a sort of epistemic vacuum. Probabilistic assignments are *always* relative to the agent's set of degrees of belief. So, strictly speaking, all probabilities which take part in Bayes' Theorem are conditional ones. Bayesians usually omit the general term to refer to this factor just to simplify the formulae, so that the role played by our background knowledge is taken for granted.

Perhaps Zalabardo could reply that what he has in mind is that the initial probabilities that appear in *PR* – *p* (*H*) and *p* (*E*) – are not well defined in comparison to the conditional probabilities – the likelihoods: *p* (*E*|*H*) and *p* (*E*|¬*H*) – included in *LR*, even though we understand them as conditional probabilities with respect to the background knowledge.[9] But that claim would still be highly controversial. Let me pause on this.

Zalabardo's paradigm for inferential knowledge is clinical testing. In those situations the values for the likelihoods *p* (*E*|*H*) and *p* (*E*|¬*H*) are usually taken from relative frequencies discovered through empirical research −the true-positives and the false-positives rates of the test. But here I cannot discern a substantial difference between likelihoods and priors. After all, what is required to ascertain those values are empirical data about frequencies. And obtaining the relevant information for the priors − the rates of asthma and lung cancer in the population − does not involve radically different procedures to those developed concerning likelihoods. So there is no reason why priors cannot be as well defined as likelihoods in contexts like these.

It should be noticed also that this is the simplest situation, since both sample spaces contain just two rival hypotheses – i.e.: asthma/¬asthma, lung cancer/¬lung cancer. But things may get much worse in more complex situations, and not only for the priors. Indeed, likelihoods may be very difficult to obtain when a sample space contains several competing hypothesis: H_1, H_2, H_3,…, H_n. In theoretical contexts where hypotheses are not formulated in a parametric format, very often we cannot be sure that the extant alternatives form a *partition* of the sample space. A condition for that is that the alternatives exhaust the possibilities so that the sum of all their respective probabilities (which are, indeed, conditional probabilities with respect to the background, as I said before) equals one. That is the reason why calculations must include the so-called *catch-all hypothesis*. Then, the sample space is $\{H_1, H_2,\ldots, H_n, H_c\}$, where H_c is the negation of all the remaining serious alternatives: $H_c \equiv \neg(H_1 \cup H_2 \cup .... \cup H_n)$.

Now, calculation of the initial probability for the catch-all hypothesis is simple, provided that we know the remaining priors, that is, $p\ (H_c) = 1 - (p\ (H_1) + p\ (H_2) +\ldots+ p\ (H_n))$. But the likelihood of the *catch-all* hypothesis may be much more difficult to obtain. Firstly, the value for $p\ (E|H_c)$ is not determined by the likelihood values of $H_1, H_2,\ldots, H_n$, in contrast to what happens with the priors, so we are bound to ascertain $p\ (E|H_c)$ separately. On the particular context chosen by Zalabardo – where the catch-all hypothesis equates just to $\neg H$ and we have information about empirical frecuencies – it is easy to ascertain the values for $p\ (E|\neg H)$. But it is not so easy when $p\ (E|\neg H) \equiv p\ (E|H_c)$ and $H_c = \neg(NM \cup TGR)$, for instance, where NM = Newtonian Mechanics and TGR = Theory of General Relativity. Unfortunately, H_c is no more than the negation of all the other possibilities. It has no genuine content. The question is how could we ascertain the probability of a particular bit of evidence E given H_c, that is, given that all the alternatives devised to account for E are false. Although this question makes full sense, it is difficult to ascertain how we could get a reasonable answer to it in some contexts.[10]

It must be acknowledged that prior probabilities have been a perennial matter of discussion among Bayesians and there is still a lively debate on it.[11] It cannot be defended, however, that likelihoods are, by and large, better defined than prior probabilities. In fact it seems very difficult to deal with an essential factor for *LR* as the likelihood of the catch-all hypothesis. My conclusion, then, is that there is no advantage for *LR* over *PR* concerning their respective coverage field on account of the fact that the former considers only likelihoods and disregards prior probabilities.

(b) *High Probability Should Not Underrate Degree of Confirmation*

Zalabardo points at the difficulties of probability measures of confirmation (*PD* and *PR*) with very probable items, in contrast to what happens with *LR*, namely, (i) very probable hypotheses cannot be confirmed to any substantial degree, and (ii) very probable evidence will not be able to provide much support for any proposition (*SRB*, 83). However, I do not consider these consequences as a shortcoming for measures of *incremental* confirmation. Let us begin with very probable hypotheses.

It seems rather plausible that the confirmational effect of evidence on a very probable hypothesis cannot be very strong. If the baseline – the prior probability of H according to the background knowledge, E discounted − is high, then the distance to the maximum value for $p\ (H|E)$ is shorter. It is precisely this distance – which can be measured in different ways, certainly – that *incremental* confirmation is concerned about. As a consequence, the very same experimental data could have greater confirmational effect on a less probable hypothesis. That is true, but incremental confirmation is a two place relation and, again, it partly depends on the point of departure, that is, *H*'s prior.

Regarding (ii), I would say that intuitions go in line with this consequence of *PR*. Unexpected events predicted by a particular hypothesis give stronger intuitive support to it than widely known experimental data. A highly improbable bit of evidence is that one which we could hardly had imagined unless we had considered seriously the hypothesis at issue so that, according to our background knowledge – which includes the extant alternatives to *H* –, *E* is really novel. Now, if *E* occurs, our confidence in *H* is notably increased. Although there is a still unsettled debate on the supposed advantages of prediction over accommodation, *PR* does justice to the intuitive bonus for novelty: the more unexpected the evidence predicted by the hypothesis, the stronger the support provided by the former – and conversely for very probable evidence.

But couldn't it occur that very probable evidence gave a great amount of support? Think about a situation where *H* and *E* are both highly probable. For instance, $p\ (E|H) = 0.99$, $p\ (E|\neg H) = 0.0099$, $p\ (H) = 0.9$. Then, $p\ (E) = p\ (E|H) \cdot p\ (H) + p\ (E|\neg H) \cdot p\ (\neg H) \approx 0.892$. According to *LR*, *E* gives a notable support to *H* while *PR* does not agree with this –notice that $LR = 100$ and $PR \approx 1.1$, which is slightly above one, the value for neutral evidence. Which option is the right one here? We could say that *E* confers greater support to *H* than to $\neg H$, sure –in fact $\neg H$ is disconfirmed by *E*. But we could also add that the confirmational impact of *E*, namely, the increase in respect of *H*'s prior, cannot be high simply because *H* was very probable before obtaining *E*. On the other side, if we keep the same values for likelihoods but interchange those of priors, the value for *LR* is exactly the same but $PR \approx 5.26$. That may be a great increment when compared to a previous value of 1.1, certainly, but recall that according to *PR* the degree of support is constrained by *H*'s prior.

Bayesian confirmation measures differ about their prior-sensitiveness [Fitelson (2007), fig. 1, and Iranzo and Martínez de Lejarza, (2010)]. The question, as I see it, is to what extent could we consider *LR* a measure of *incremental* confirmation insofar as the likelihood ratio is the only factor which determines it. *LR* satisfies the Bayesian Relevance Criterion aforementioned, since $p\ (H|E) >/=/< p\ (H)$ *iff* $\frac{p(E/H)}{p(E/\neg H)} >/=/< 1$. It must be accepted, then, that incremental confirmation not only obtains when the true-positives rate exceeds that of false-positives, but it also demands this surplus. It could also be said, notwithstanding, that what *LR* measures is not so much incremental confirmation as comparative evidential support in respect of the *catch-all hypothesis*.

(c) *Deductive Support Amounts to Infinite Support*

When $\neg E$ is a deductive consequence of $\neg H$, $p\ (\neg E|\neg H) = 1$ and $p\ (E|\neg H) = 0$. In that case *LR* is undefined, no matter what the value for $p\ (E|H)$ might be. Nonetheless, Zalabardo encourages us to consider deductive support as a "limiting case" of non-deductive support where *E infinitely supports H*. After all, if a and b are real numbers included in the interval [0, 1] and $a \neq 0$, then $\lim_{b \to 0} \frac{a}{b} = \infty$.

My first modest point about this is that deductive contexts may be highly misleading. Let us think about a particular example with a standard dice (i.e., six faced and non-loaded). *LR* tells us that we should attribute an infinite degree of support in case that $H = E$, a clear case of reciprocal deductive entailment. But claiming, for instance, that "Obtaining an even result" infinitely supports "Obtaining either 2 or 4 or 6" hardly makes any sense, on my view. We should grant, at least, that the kind of support we are talking about in a claim like this, if any, is very different from that involved in standard discussion on confirmation, and not just "a limiting case". The situation is not much better for *PR*, to be sure. According to it "Obtaining 6" confirms "Obtaining a number higher than 5" to a greater extent than "Obtaining either 2 or 4 or 6" confirms "Obtaining an even result". In fact, *PR* = 6 and *PR* = 2, respectively, although this difference regarding the degree of support can hardly be justified.

Many more unpleasant consequences could be found when deductive relations are involved. On account of the foregoing, I am not persuaded that explicating deductive entailment, as a limiting case or by some other expedient, is a target for incremental confirmation. Deductive contexts suggest possibilities very distant from the typical domains where confirmational measures are intended to apply.

Leaving aside deductive relations, the fact is that *LR* could yield an infinite value in some empirical situations. Zalabardo himself alludes elsewhere to a test with no false-positives. In medical contexts, the *sensitivity* of a test is the proportion of people that have the disease and test positive for it, that is, $p\ (E|H)$, while the *specificity* of a test is the proportion of people who does not have the disease and test negative for it: $p\ (\neg E|\neg H)$.[12] Hence, Zalabardo's example appeals to a test with maximal *specificity*, since $p\ (E|\neg H) = 0$ (no false-positives) entails that $p\ (\neg E|\neg H) =1$. The specificity of a medical test is an empirical question and, even though it is improbable that actual, non-hypothetical, frequencies with a high number of trials give no false-positives, it may occur. However, it is worth noticing here, firstly, that maximal specificity is not an incidental possibility for Zalabardo – in fact, his argument on the objective conditions for inferential knowledge crucially depends on this point. In particular, the reason why a high value for $p\ (E|H)$ is discarded as a necessary condition for adequate support is that a very low value for p ($E|H$) would not prevent E from providing adequate support for H *when the clinical test has virtually no false-positives* (*SRB*, 75; my emphasis).[13] And secondly, it is precisely in those non-deductive contexts where confirmation measures are expected to work successfully. Now, suppose that empirical frequencies about symptoms and diseases from a database of clinical examples yield the following values:

$p\ (E|H) = 0.5 \qquad p\ (E|\neg H) = 0$

$p\ (E^*\ |H^*) = 0.5 \qquad p\ (E^*\ |\neg H^*) = 0.01$

We could assume that the remaining relevant factors are neutralized – both samples are of similar size and have been obtained through random sampling, the priors for H and H^* are similar,… In order to calculate *PR*, prior probabilities should be known. If p (H) = p (H^*) = 0.05, for instance, *PR* (E, H) = 20 and *PR* (E^*, H^*) ≈ 14.50. So, a consequence of *PR* is that H is more confirmed by E than H^* is confirmed by E^*. That seems fine.

Let us see now how *LR* copes with this example. *LR* (E^*, H^*) = 50, but *LR* (E, H) is either undefined or infinite? Resorting to intuitions, it seems clear that, other things equal, E confirms H to a higher degree than E^* in respect of H^*. As a consequence it could hardly be accepted that *LR* (E, H) is undefined. We would say, rather, that whatever it could be, it would be higher than that of *LR* (E^*, H^*). Alternatively, if we defend that E infinitely supports H, we also endorse the claim that E's positive confirmational effect on H is infinitely higher than that of E^* on H^*. But, is that tiny difference about the rate of false-positives enough for claiming such a huge difference between *LR* (E^*, H^*) and *LR* (E, H)? Furthermore, it seems a bit surprising that empirical research, where only discrete variables are involved, could yield an infinite value for confirmational support.

The moral of this section is that intuitions do not favour *LR*, qua measure of incremental confirmation, over *PR*. Certainly, some further criteria have been invoked to settle the issue of the plurality of confirmation measures. Recent experiments in psychology of reasoning, for instance, have been adduced in favour of an alternative to *PR* and *LR* [Tentori *et al.* (2013)]. And there are also pluralists who accept that different measures could be chosen depending on the context [Steel (2007), Joyce (2008)]. Scrutinizing those additional criteria is beyond the purview of this paper. My contention is that Zalabardo's preference for C2* is unjustified *as far as it is based on LR's intuitive appeal*.

At this point of our discussion someone could suggest that maybe incremental confirmation is irrelevant for adequate support. But recall that conditions (C1) and (C2) are not sufficient for adequate support and that is the reason why incremental confirmation is an unavoidable additional requirement for Zalabardo. Although I quite agree with this, I also think that a “confirmational analysis” of adequate support could avoid commitment with any of those particular measures of incremental confirmation. In the next section I will explore this possibility while keeping in mind – I hope – the main insights of Zalabardo’s proposal.

III. ADEQUATE SUPPORT AND INCREMENTAL CONFIRMATION

It should be noticed that objective conditions for adequate support, i.e.: C1 and C2*, guarantee that *E* is relevant evidence for *H* insofar as C2* entails that $p\ (H|E) > p\ (H)$. However, this pair of conditions do not distinguish between “$p\ (H) < b < p\ (H|E) < 1$” and “$b < p\ (H) < p\ (H|E) < 1$” even though there is a non-negligible difference between both situations. In the first one the boundary stated by *b* is crossed and that seems essential for *E* providing knowledge of *H*. The second situation, in contrast, reports an increase in probability that may be highly desirable but it is a gain just in surplus support. Although in both situations *E* is evidentially relevant to *H*, insofar as it raises *H*’s probability, *E* does not play the same role with respect to inferential knowledge. Conditions C1 and C2* tell us that when we inferentially know *H* on account of some evidence *E*, *H*’s probability is boosted above *b* on account of *E*. Now, getting some further evidence *E*’ in favour of *H* could raise its probability even more, but it is not clear that this additional evidence provides knowledge of *H*. Actually, taken for granted that the subjective conditions for knowledge are satisfied, we (inferentially) know *H* before getting this additional evidence *E*’. We could say that *E’* incrementally confirms *H* and, consequently, we are more confident about *H* now than before. But it could hardly be said that *S* inferentially knows *H* on the basis of *E’*, if *H* is already known on account of *E*, *even though C1 and C2* are met*.

According to this, I suggest some modifications on Zalabardo’s objective conditions for adequate support as follows:

C1 $0.5 < b < p\ (H|E) < 1$ (where *b* is a real number higher than 0.5)

C2 a low value for $p\ (E\ |\neg H)$

C3 $p\ (H) < b$

According to the argument developed in the preceding section, condition C2* – a high likelihood ratio – is eliminated and a step backward is made for recovering C2. C1 is preserved and a new condition C3 is added.

The rationale for C3 is twofold. It guarantees that *E* is relevant evidence for *H* since it follows from C1 and C3 that $p\ (H|E) > p\ (H)$. Evidential relevance is Zalabardo's concern to defend C2*, but C3 can also do the job with no commitment to any particular measure of incremental confirmation. Consequently, there is no need to appeal to dubious intuitive advantages. Furthermore, by adding C3 it is demanded that *E* has such a positive confirmational impact on *H* that the probability of *H* is raised above the threshold *b*. Thus, adequate support for *H* is obtained only when the evidence at issue raises the probability of *H* above *b*. Recall the litmus paper test. Let us suppose that the test satisfies C2 and also that we have no clue about whether the solution is an acid or not, so, in principle, p (ACID) = p (NON-ACID) = 0.5. If litmus paper turns red when dipped, p (ACID | RED) > 0.5 > p (NON-ACID | RED). But (RED) is adequate evidence for inferential knowledge of (ACID) only insofar as, in virtue of (RED), the probability of (ACID) is above the threshold *b*. The next day I dip again the litmus paper in the solution and it turns red, I am more confident about (ACID), its probability has raised, indeed but, even though conditions C1 and C2* are met, I do not get knowledge of (ACID) since I knew it before dipping the paper at the second time. Keeping in line with this, conjunction of C1 and C3 entails that in those situations there is no adequate support *for inferential knowledge*.

Turning now to C2 –a low rate of false-positives–, the rationale for it is that *E* cannot be adequate evidence for *H* unless it would be unlikely to obtain when *H* is false. Even though C1 and C3 jointly entail that $p\ (E|\neg H) < p\ (E|H)$, I agree with keeping C2 as a condition for adequate support. Here is an example: $p\ (E|H) = 1$, $p\ (E|\neg H) = 0.6$ and $p\ (H) = 0.4$. Then, $p\ (H|E) \approx 0.53$. Let us suppose that *b* is slightly above 0.5, so $0.5 < b < 0.52$. The degree of incremental confirmation provided by *E* is not high but it is enough to boost *H*'s probability above *b*. C1 and C3 are fulfilled but would we consider that *E* provides adequate support for *H*? It is not easy to give a yes/no answer to a question like this. When it is not easy to gather further evidence and the decision between *H* and ¬*H* is pressing, perhaps we would be prone to an affirmative answer. But *E* does not seem a reliable indicator of *H*. It does not adequately protect us against the error of taking as true what is false ("Truth tracking is, first and foremost, sensitivity", *SRB* 56). In fact, it may confuse us since it is not unlikely to occur *E* and ¬*H* jointly These considerations play in favour of keeping C2 among the conditions for adequate support and, a fortiori, for inferential knowledge.

Inferential knowledge necessarily involves adequate evidential support. I also subscribe to the idea that evidential support is enlightened by a quantitative account of confirmation. As a result, incremental confirmation – i.e.: satisfaction of the Bayesian Relevance Criterion – is crucial. But we can discern that *E* has boosted the probability of *H* to the desired threshold *b* just by *comparing* $p\ (H)$ and $p\ (H|E)$. That is the point for subscribing C1 and C3. Objective conditions for paradigmatic cases of inferential knowledge are not exhausted by C1 and C3 –recall that condition C2 should be preserved. However, I take the set of conditions (C1 & C2 & C3) as an improvement on Zalabardo's favoured set – that is, C1 & C2* – since (i) situations where relevant evidence does not give, after all, substantial support for (inferential) knowledge are discarded, and (ii) commitment with a particular confirmation measure − based on its alleged intuitive advantages − is circumvented.

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NOTES

[1] For a similar condition regarding a different sort of inferential knowledge – i.e.: "inference to the best explanation" − see my (2007), sect. 4.

[2] These are the main alternatives, certainly, but many more have been suggested. See Crupi *et al.* (2007) for a comprehensive list.

[3] Sometimes ordinal inequivalence arises when we compare the incremental confirmation afforded by *E* to rival hypotheses, so that depending on which measure is used the hypothesis most favoured by *E* will vary. Sometimes it can be detected in more complex contexts where non-rival hypotheses are compared in respect of different bits of evidence [Fitelson (1999), (2001), Crupi *et al.* (2007), Iranzo and Martínez de Lejarza (2010)].

[4] I will leave aside difference measures. I do not think that they can be successfully defended [see Iranzo and Martínez de Lejarza (2010)].

[5] The first equivalence follows directly from Bayes' Theorem: $p(H|E) = \frac{p(E|H)p(H)}{p(E)}$. The second equivalence is a consequence of the Theorem of Total Probabilities.

[6] For a thorough discussion on the difference between *PR* and *LR* concerning their axiomatic foundations, see Crupi *et al.* (2013).

[7] The general form for the theorem is "If likelihoods for *H* and *H** are even and evidence (*E* and *E**, respectively) is confirmatory (disconfirmatory), then *PR* favours that hypothesis with the lowest (highest) prior". See the proofs at the appendix of Iranzo and Martínez de Lejarza (2013).

[8] When uneven priors go on stage – precisely those situations where *LR* and *PR* may diverge, as I argued before – intuitions get unclear. Let us suppose that we have good independent grounds to think that it is very unlikely that the patient suffers from asthma. We also have some previous information that makes likely that she has lung cancer. Wouldn't we tend to think that she is one of those few false-positives about wheezing and asthma? Conversely, wouldn't we seriously consider that she is a true-

positive example about loss weight and lung cancer, in spite of the non-negligible rate of false-positives? If that is the case, would we still say, with no more qualifications, that wheezing confirms asthma to a higher degree than loss weight confirms lung cancer as Zalabardo claims? Intuitions turn unclear when prior probabilities are considered and they go in opposite directions with respect to likelihoods. Consequently, the intuitive support for *AC* –and also for *LR*– is weakened. [For further details see Iranzo and Martínez de Lejarza (2013).]

[9] Sherrilyn Roush agrees with Zalabardo on his reluctance to priors –albeit for different reasons. She develops a tracking account of evidence where the priors of the hypotheses are understood as dependent variables. The independent variables are *LR* and p ($E|H$). She proves that when p (E) is fixed, the posterior probability of the hypothesis p ($H|E$) is determined. However, she insists that a high value for p (E) is a desideratum − a sufficient but not a necessary condition − for E being evidence for H. Thus, her disdain for priors is specially directed towards those of the hypotheses − even though calculation of p (E) necessarily involves p (H). [Roush (2005), chap. 5 and appendix 5.1.]

[10] Salmon (1996) illustrates the difficulties with "negative likelihoods". For a less pessimistic opinion, see Roush (2005), 211 and ff.

[11]There are many suggestions to implement probability laws with some constraints on prior probabilities [Weisberg (2011)]. So-called "objective" Bayesians are gaining popularity in recent times. See also my (2008) and (2009).

[12] *Sensitivity* and *specificity* are respectively related to the test's ability to identify positive and negative results. It should be added here that, in statistical test theory, a false-positive result is a "Type-I error" and a false-negative result is a "Type-II error". Then, maximal specificity equates to zero Type-I error rate – indeed, Type-I error = (1 – *specificity*) and Type-II error = (1 – *sensitivity*). Incidentally, a null value for Type-I error rate is rather unusual in real contexts of *statistical* hypothesis testing.

[13] See also the discussion on *sensitivity* on account of Nozick's *one-sided methods* (*SRB*, 58 and ff.)

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RESUMEN

Una vez explicado el conjunto de condiciones que Zalabardo exige para el conocimiento inferencial, argumento que una de esas condiciones es defectuosa. Una dificultad adicional se considera en la sección III. Para hacerle frente, sugiero algunas modificaciones de la propuesta de Zalabardo que preservan, no obstante, su intuición básica

PALABRAS CLAVE: *conocimiento inferencial, confirmación, evidencia, soporte evidencial, fiabilismo.*

ABSTRACT

After explaining Zalabardo's set of objective conditions for inferential knowledge, I will argue that one of those conditions is seriously wanting. A further difficulty for his account will be considered in section III. In order to cope with it, I will suggest some modifications on Zalabardo's proposal while preserving its main insight.

KEYWORDS: *Inferential Knowledge, Confirmation, Evidence, Evidential Support, Reliabilism.*