Agustín Rayo, *The Construction of Logical Space*, Oxford University Press, Oxford, 2013, 220 pp.

I like this book. You should read it. Especially if you care about what's going on right now in phil math, metaphysics, applied logic, or phil language. And if you're into ontology or modality specifically, you'll need a copy *tout de suite*.

A lot's packed in to these 200-some pages, but I think it's fair to say that *one* of Rayo's main aims is to establish the existence of numbers (and other mathematical entities). Or more modestly, to clear a new path to Platonism for fellow travelers: those of us antecedently equipped with a permissive, pragmatic attitude toward ontological commitment. He calls his version of the view Trivialist Platonism (TP), because it says that arithmetic is not just true (and not just necessary-given-the-existence-of-its-objects), but downright tautological.[1]

Oddly, as far as I can tell, Rayo's book doesn't actually contain an argument for TP. Which is not to say it's just a manifesto. Tonally, it reads more like "a day in the life" of a trivialist. And more substantively, I should stress that Rayo does provide interesting arguments for plenty of stuff in TP's vicinity. The most important case in point is his argument that numbers exist, to which I devote most of my attention in the pages below.

Let's begin at the end. Rayo's case for countenancing numbers culminates in a simple proof (p. 75), premised on the following schematic principle (repeated koan-like throughout the book):

> NUMBERS (N): For the number of the Fs to be *n just is* for there to be exactly *n* Fs.

N basically expresses an equivalence between using noun-numerals (on the left) and using adjective-numerals (on the right). The instance Rayo needs for the proof results from replacing '*n*' with '0' and replacing 'F' with 'number'. Actually, all he needs is a material biconditional weakening of it —i.e., that the number of numbers is 0 iff there are 0 numbers. (More on the 'just is' operator soon.)

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[1] So aficionados could call him a neologicist. But don't call him a neo-Fregean. (See section 3.2.) Rayo does delineate a trivialist version of neo-Fregeanism. But he also points out (3.2.2) that trivialists needn't fetishize Hume's Principle.

Suppose for *reductio* that there were 0 numbers, by which we just mean: $\sim\exists x Num(x)$.[2] Using our biconditional, we can infer that the number of numbers would be 0. But if the number of numbers were (identical to the number) 0, then there would be at least one number, namely: 0. So the supposition that there aren't any numbers is self-undermining, given N.[3]

If you're willing to invoke additional premises, you can argue for numbers more directly. Say there are no dinosaurs, to use Rayo's favorite example. Or if you'd prefer a tautology: that nothing is distinct from itself. Applying the relevant instances of N, you can prove twice over that 0 exists. (And if there's no God but God, you secure 1; etcetera.)

Unfortunately, both versions of the argument can be challenged on Alstonian grounds. Like a synonymy claim or biconditional, N is meant to be symmetric (p. 5). But as Alston famously pointed out, this sort of symmetry suggests that the Platonist and the nominalist are actually on equally footing.[4] That the number of Fs is 0 *seems* to entail the existence of numbers; that there are no Fs *seems* not to. The Platonist thinks we should go with the first appearance and conclude that both claims do. The nominalist thinks we should go with the second appearance and conclude that neither does.

Rayo might reply that the left-hand side of N should be regimented as an existential quantification (rather than an atomic identity statement) —i.e., for there to exist an x such that x is the number of Fs and x is n.... Thus our "first appearance" would be harder to resist. But this move only makes N seem more substantive. Why should we believe it?

Chapter 1 is effectively a preemptive, defensive maneuver to block one reason for rejecting N out of hand. Chapter 2 then presents a general account of when we ought to accept a given 'just is' statement. And in the opening pages of Chapter 3, Rayo argues that

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[2] More generally, 'There are exactly n Fs' is regimented in the language of applied arithmetic using an unbounded existential quantifier and the identity sign. For example, 'There are exactly two Fs' becomes: $\exists x\exists y(Fx\&Fy\&x \neq y\&\forall z(Fz \to (z = x \text{ or } z = y)))$. More concretely: to say I have exactly two dogs is just to say that there are C and R such that each is a dog of mine, they are numerically distinct, and any dog of mine is identical to one of them.

[3] We can also get that the existence of numbers is a logical truth if we assume, with Rayo, that the 'just is' operator expresses logical equivalence. But we can't recover all of Peano arithmetic from N and logic alone. Which is why I say the book doesn't contain an argument for TP.

[4] W.P. Alston (1958), "Ontological Commitments", *Philosophical Studies*, vol. 9, nos. 1–2, pp. 8–17

this account recommends N in particular. I'll work through these moving parts in order, and then wrap up with some comments on the semantics for arithmetical discourse offered in section 3.3.[5]

Rayo eventually offers a bunch of different ways to think about the 'just is' operator, but let's follow his lead and begin with a fairly fuzzy, intuitive idea. N basically says that its two sides provide "full and accurate descriptions of the same feature of reality", as Rayo often puts it. To take one of his less contentious examples: the sentences 'Susan is a sibling' and 'Susan shares a parent with someone' arguably pick out the same state of affairs (assuming they're true).

Anxious metaphysicians might worry that these two sentences can't possibly pick out the same state of affairs, since they have different logical forms. (Let's suppose the first is a monadic predication in LF; and the second, a doubly quantified conjunction of dyadic predications.) After all, a sentence "picks out" whatever state makes it true, and truth is a matter of correspondence to reality, and correspondence involves structural isomorphism, and a single state of affairs can't have two (total) structures, right? Right?!

This breathless string of assumptions is roughly the philosophical picture Rayo calls metaphysicalism. Metaphysicalism obviously obstructs his path to Platonism, given how we're thinking about N's syntax. Now, as a deflationist about truth, I'm more than happy to set metaphysicalism aside. But Rayo doesn't explicitly discuss the debates between deflationists and correspondence theorists.[6] What he does say about truth in subsequent chapters suggests a simple correspondence *sans* structural isomorphism view. (More on this below.) But I don't think those brief remarks are meant to constitute an independent argument against metaphysicalism.

I imagine some people would want to reject metaphysicalism on the ground that a single state of affairs *can* have more than one metaphysical structure. But Rayo repeatedly demurs from using the notion of metaphysical structure to do any philosophical work. Maybe he means to make an exception for "defensive" work like shifting the burden of proof back onto the metaphysicalist. I'm not sure. But

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[5] I'll also point to supporting passages from Chapters 5 and 7 along the way—passages on primitive modality and nominalist paraphrase, respectively. Unfortunately, I won't have space to say anything about Rayo's epistemology of mathematics (Chapter 4) or his favorite version of actualism (Chapter 6), despite their intrinsic interest.

[6] Insofar as the early Wittgenstein was a correspondence theorist and the later Wittgenstein was a deflationist, section 1.2.2 is an *implicit* discussion of these debates. But it goes by very quickly.

in any event, I have a hard time seeing how one state could have two separate structures. Compare bridges, which come in a variety of structures: beam, truss, cantilever, suspension, etc. One can of course construct unsightly hybrids, but there's no such thing as a pure cantilever bridge that doubles as a pure suspension bridge.

Rayo's case against metaphysicalism is much less direct than the argument from deflationism or the argument from multiple structures. The dialectic of Chapter 1 suggests that Rayo simply thinks a competing philosophical picture is ultimately more compelling than metaphysicalism. He calls his alternative compositionalism. It's a combination of two views: that a singular term 't' refers to something (or other) if the sentence 't exists' is true (p. 15); and a more complicated claim about what exactly being a singular term amounts to (p. 14). Now, I expect some of Rayo's readers would complain that compositionalism hasn't been adequately supported. But I'm actually having the opposite problem:

Compositionalism seems trivial to me, and thus compatible with metaphysicalism. Take its first component, linking singular reference and existential truth. Insofar as this is just a material conditional, I can't imagine who would deny it.[7] (In particular, for any *reductio* of the conjunction of metaphysicalism and compositionalism that uses the conditional, there should be an equally compelling argument exclusively concerning existential truth.) A more substantive view would be that *what it is* for 't' to refer is for 't exists' to be true. But I for one don't see why referential success should be contingent on the presence of quantificational vocabulary in the language at issue. Anyway, Rayo doesn't advance the stronger view. And he clearly wants to reject the related suggestion that for 't' to refer *just is* for t to exist (p. 33).

The compositionalist's theory of singular-term-hood is admittedly a bit less trivial, but it doesn't seem to generate a conflict with metaphysicalism either. When Rayo tries to explain the alleged conflict (pp. 16–17), he appears to import two additional premises. First: we can simply *stipulate* that a sentence with one LF —from an antecedently uninterpreted artificial language— has the same truth-condition as an old sentence with some other LF. And second: sentences with the same truth-condition pick out the same state of affairs. But the conjunction of these premises is already incompatible

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[7] Of course, it will have to be refined for context-sensitive terms, but Rayo is mainly interested in the language of mathematics. See note 4 in Chapter 1.

with metaphysicalism. So compositionalism is moot, and Rayo's argument seems question-begging.

In sum: I happen to agree with Rayo that the LF of N is no obstacle to its truth, but his own reasons for thinking so seem inadequate to me.

Supposing for the sake of argument now that logical form is irrelevant, how should we decide whether or not to accept a given 'just is' statement? In a representative nutshell (p. 19), Rayo's answer is: "One has to balance the cost of rejecting the relevant statement —an increase [in] the range of questions that are regarded as rightfully demanding answers— with the cost of accepting the statement —a decrease in the range of theoretical resources one has at one's disposal." The bulk of Chapter 2 is dedicated to fleshing this answer out. I won't rehearse Rayo's examples, but you can imagine how standard arguments for and against, say, physicalism (regimented as a 'just is' statement) might be massaged to fit this cost/benefit mold.[8]

Rayo's answer is odd. I'd have thought we should accept a 'just is' statement just in case... *it's true!* Or in a more subjective register: just in case we can marshal sufficient evidence for it. Rayo does think such statements are apt for truth and falsity. He's not an expressivist about 'just is' statements.[9] So it's *prima facie* puzzling why our epistemology here should be so pragmatic —so steeped in cost/benefit considerations. It doesn't seem like Rayo subscribes to a general, pragmatist theory of truth or epistemic justification.

The closest he comes to articulating a general conception of truth is the following (pp. 57–58, original emphases): "To *set forth* a statement is to make a distinction amongst ways for the world to be —to divide logical space into distinct regions— and to single out one side of this distinction; for the statement to be *true* is for the region singled out to include the way the world actually is." In other words: a sentence is true just in case the actual world is a

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[8] Rayo says rejecting physicalism raises embarrassing questions (p. 21). In other words: the dualist has more explaining to do, because identity claims nip certain explanatory demands in the bud. To pick a nit: this seems like a misleading way to set up the dialectic. After all, the main problem for physicalism is the explanatory gap. More generally, when the main thing to be explained is the appearance of distinctness, identities don't help!

[9] Though he floats the kindred view (2.3.2) that they're really claims about what words/concepts (we ought) to use for representing the world (rather than claims about the world itself). A general expressivism about normative discourse could then be used to bridge the gap. And quasi-realism might be invoked to make sense of Rayo's talk of truth and falsity for 'just is' statements. But he doesn't discuss these possibilities.

member of the unstructured proposition it expresses. In itself, this
familiar thought doesn't solve our exegetical puzzle, but it does play
an important role in the solution.

It turns out Rayo's a relativist about 'just is' statements. He thinks
they aren't true or false *simpliciter*, but only relative to one or
another "conception of logical space" (CLS) —basically, a systematic
view about what's logically possible. This relativism is rooted in
the foregoing remarks on truth. As he immediately proceeds to say
(p. 58, his emphasis): "On this way of seeing things, the notion
of truth *presupposes* a conception of logical space: the distinction
between the true and the untrue is just the distinction between
regions of logical space that include the way the world actually is,
and those that do not."

Now, Rayo needn't be a global relativist. Truth in chemistry, for
example, can be absolute so long as every CLS is silent on chemical
claims (or they all say the same thing). So why is he a relativist about
'just is' statements in particular? Because he thinks conceptions of
logical space can be individuated by maximal collections of 'just is'
statements (p. 51). A claim is logically possible (on a given CLS)
just in case it's compatible with the relevant 'just is' statements.[10]
So (p. 58): "Every conception of logical space will count those 'just
is'-statements on which it is based true, and the rest false."

Even if we grant Rayo's individuation claim, however, the most
we can conclude is that there are absolutely true 'just is' statements
iff there is an "objectively correct" CLS iff . . . there are absolutely
true 'just is' statements. I don't see an argument for relativism.

Rayo might actually agree.[11] But what we definitely disagree about
is the impact that this concession would have on his case for Pla-
tonism. Rayo claims (pp. 63–64) "that nothing in what follows will
depend on how one ends up thinking about the correctness of a
conception of logical space". But his argument for accepting N cru-
cially turns on the sort of cost/benefit calculation described above.
To paraphrase (p. 74): rejecting N and thereby opening up a logico-
metaphysical gap between substantival and adjectival uses of numer-
als raises embarrassing philosophical questions without offering any
obviously fruitful theoretical resources in return. (For the record: I

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[10] I don't think we're ever told what the relevant notion of compatibility is.

[11] As he later admits (p. 63): "For all I have said, it may well be possible to
characterize an 'objective' notion of correctness, one which is relative neither to a
conception of logical space nor to the aims of a particular community. I don't know
how to do so myself, but would be delighted if it could be done." As I'm about to
say, you can do it with states of affairs.

agree.) Now, if relativism about 'just is' statements were true, these sorts of pragmatic considerations might carry the day. But if relativism is *false*, then the real question is: is N true? So, in the absence of an argument for relativism, Chapter 2 looks otiose to me.

Here's a flat-footed reason to resist relativism about 'just is' statements. Rayo grants that we can think of the 'just is' operator as an identity predicate (p. 66), and thus that N is something near enough to an atomic identity statement. Plausibly, an identity statement is (objectively, absolutely) true just in case its two terms refer to the same thing. As we said earlier, the 'just is' statement from Rayo's sibling example will be true just in case its two sides pick out the same state of affairs. Ditto for N. Insofar as co-reference is a non-relative, fully factual matter, N should share this status.[12]

One might have hoped the rest of Chapter 3 (Mathematics) would finally provide a positive argument that N is true.[13] (The farthest we've gone so far is p. 75, its third page.) On initial read-throughs, I thought the semantics for arithmetical discourse in section 3.3 was supposed to yield some such argument. It certainly feels like the climax of the first half of the book (Main Texts). But upon reflection, I've come to think it's really just another defensive maneuver. Let me explain.

N's plausibility arguably turns in part on whether it can be construed as a fragment of some more general, systematic account of arithmetical states of affairs. N just concerns states that can be picked out using sentences of the form, 'The number of the Fs is *n*'. Can we extend N to the entire language of applied arithmetic?[14] Well, that depends on what exactly we're allowed to put on the right-hand side. If we're allowed to reuse the language of arithmetic, we can just

[12] Rayo might want to reply by distinguishing cases of co-reference to intrinsically *modal* entities from cases of co-reference to non-modal entities. He could say my absolutist intuitions, above, are just tracking the latter, so that rejecting relativism about the former would be a hasty over-generalization. Rayo does seem to think 'just is' statements have modal *relata*. We've been calling them states of affairs. But he often calls them truth-conditions (i.e., sets of worlds) as well. And much later in the book (Chapter 5, p. 135), he speculates that modality is utterly ubiquitous.

[13] Pragmatic reasons for accepting N (or accepting that it's true) could conceivably enter into such an argument. Perhaps N itself is the best *explanation* of the (putative) fact that its acceptance has the pragmatic virtues Rayo describes. But I don't think anything like this is ever explicitly suggested in the book.

[14] Applied arithmetic extends pure arithmetic with singular terms for "the number of Fs". When 'F' is an arithmetical predicate (e.g., 'number') these terms still look quite "pure". But strictly speaking, they're applied. And keep in mind that the right-hand side of N contains no arithmetical vocabulary whatsoever, pure or applied.

pair each sentence with itself. These 'just is' statements certainly ring true. But they don't include any instances of N. And they can't be used to argue (non-circularly) for the existence of numbers. Can we do better?

This looks like the good old nominalist paraphrase project in a new guise. Much later in the book, Rayo says that he doesn't find the project particularly interesting or worthwhile (though he does cite a co-authored contribution to it). His main reason seems to be that:

> any claim to the effect that a suitable paraphrase-function exists or fails to exist must be tied to a non-trivial claim about the legitimacy or ille-gitimacy of a given set of expressive resources. And it seems dangerous to rest any substantial conclusions in the philosophy of mathematics on one's views about the legitimacy of a potentially controversial piece of vocabulary. (pp. 171–172)

I'm not sure what the danger is here, or what the relevant notion of legitimacy might be. Consider for example the sort of infinitary para-phrase strategy Rayo discusses in section 7.4. At bottom, the trick is just to replace existential (universal) generalizations with infinite disjunctions (conjunctions). Granted, this gambit may not provide a realistic theory of ordinary arithmetical representation or under-standing. But that doesn't tarnish its credentials as a metaphysics of arithmetic.[15] And N is first and foremost a metaphysical view.

Anyway, getting back to Chapter 3, Rayo offers a compositional semantics for the language of applied arithmetic with the following interesting feature. The truth-condition of any given sentence is a function of the truth-conditions of sentences drawn exclusively from the *non*-arithmetical fragment of the language. The reason this recipe falls short of a nominalist paraphrase is that Rayo (recursively) spec-ifies the relevant *functions* using the language of arithmetic. For example, the semantics says that '0 = the number of zs such that z is a planet' is true at a world just in case 0 is the number of zs such that 'z is a planet' is true at that world.[16] And in the limiting

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[15] It's admittedly unclear whether the view would amount to nominalism or trivial-ism, for the simple symmetry reason rehearsed earlier. But as we said, that's already a problem for Rayo.

[16] So you don't quite recover N itself. But if you assume a material biconditional version of it in the metatheory, you can derive every instance of the following from the semantics: 'The number of the Fs is n' is true at w iff there are exactly n zs such that 'z is an F' is true at w. In this special case, then, we don't have to use arithmetical vocabulary to specify the truth-condition of the original sentence.

case of pure arithmetic, the semantics delivers a constant function. In other words, the truth of a sentence of pure arithmetic (at a world) does not depend on the truth of any other sentence (at that world). '1 + 2 = 3' is true at w just in case $1 + 2 = 3$, period. Under the assumption that arithmetic is true, the semantics effectively tells us that it's necessary.

What exactly is the philosophical value of this semantics? Rayo doesn't claim it provides justification for believing N. After all, it's just a systematic specification of the truth-conditions *trivialists* want to assign arithmetical sentences.[17] (And thus, as I would spin it: a prophylactic against the generality concern a few paragraphs back.) But as he himself admits earlier, you can accomplish much the same thing by simply saying that what it is for the axioms of arithmetic to hold is just for a tautology to hold.[18] The main advantage he seems to claim for his semantics is that it's compositional and therefore "significantly more illuminating" (p. 79). But I for one don't see what's illuminated. Compositional semantics may shed light on semantic competence, but Rayo isn't trying to explain our knowledge of the language of arithmetic.

So, suffice it to say, there's a lot Rayo and I disagree about. Ironically, I actually tend to think N is true, and thus that there are numbers. Which is why I've been scouring the book for proof. At the very least, reading Rayo has helped me clarify my own thinking on the subject tremendously. I can't remember the last philosophy book that was so rewarding to work through.[19]

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[17] As he emphasizes, even non-trivialists can accept this claim (*about* trivialists); unlike with a homophonic semantics.

[18] Of course, this doesn't give you truth-conditions for contingent sentences of applied arithmetic. But so what?

[19] Many thanks to Rayo himself for setting me straight on a number of interpretive, philosophical, and technical issues *en route* to this review; and for valuable feedback on an earlier draft.